

Research Letter

The Significance of the Kampé de Fériet functions in the Computation of Certain Generalized Distributions

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This journal letter is a summary of the research that I performed in the past six years from 2016 until 2021 and it summarizes the significance of the Kampé de Fériet functions and of the Srivastava functions in the computation of the generalized distributions such as the generalized Bessel function distributions and of the generalized parabolic cylinder function (or GPCF) distributions. The main goal of this journal letter is to emphasize that the research I performed and published from 2016 until 2021 is *independent, original, unique, novel, and innovative*. While this research may have some similarities with one particular example which is the Bessel Function Distribution of the first kind that is a special case of the incomplete Lipschitz-Hankel integrals of Anger and Weber functions.

Index Terms—Generalized parabolic cylinder function, GPCF, generalized Bessel function, Kampé de Fériet function, independent, original, unique, novel, innovative, distributions.

1 Introduction

The main purpose of this journal letter is to set the record straight and touch on (or specify, identify, mention, or indicate) that without a single (or slightest) doubt Dr. Progri's research work between 2015 and 2011 (see Progri (2016, [1])-(Progri (2021, [9])) is *independent, original, unique, novel, and innovative*.

Why was the research and Dr. Progri performed in *independent*? Since the research that I performed was self-sufficient, self-supporting, self-sustaining, self-reliant, self-constrained, and self-made it was therefore entirely independent research that included analysis, modeling, simulation, computation, verification, peer-review, and publication.

Why was the research and Dr. Progri performed in *original*? The originality of the research has to do with two main aspects of the research: being the first in time or among the first and

being authentic (i.e., not a derivation from someone else).

In order to successfully address and understand the originality of the research that Dr. Proгри performed we have to understand the volume of the research that I performed from 2016 until 2019 that was dedicated to three journal papers only on the computation of the generalized Bessel function distributions Proгри (2016, [1]), Proгри (2018, [3]), and Proгри (2019, [4]) and four journal papers only on the computation of the GPCF distributions Proгри (2016, [5]), Proгри (2021, [7]-[9]). The volume, content, and duration of the research has only one explanation that the research that Dr. Proгри performed was original.

Was this research work *authentic*? I.e., not copied, genuine, actual, real, true, bona fide, etc. The utilization of the Ithenticate tool or anti-pilgrim (or similarity) software verified that the research that Dr. Proгри published from 2016-2021 was his and his alone. There was one example; however, when Dr. Proгри was asked to review another journal paper that showed some similarities with another author Millerⁱ (1998, [13]) in the case of the computation of the Generalized Bessel function distributions Proгри (2016, [1]), Proгри (2018, [3]),

As a reviewer I had a limited role; i.e., it is not the job of a reviewer to write a journal paper; i.e., it was not really my job as a reviewer to show the similarities and differences of Proгри (2016, [1]), Proгри (2018, [3]), and Proгри (2019, [4]) the Miller's work ([11]-[15]). Nevertheless, a few years later I felt compelled to write his letter and address and ambiguity that the publication of the journal paper I peer-reviewed may have risen.

Was the research that Dr. Proгри performed *novel*?

The word *novel* means: "If it's a new knowledge or expansion to already existing one, original, not in the literature, and done under research ethics, then it is a novel research work."

In seven publications that Dr. Proгри published from 2016 until 2021 only two publications contain two equations that have some similarity to the work that I performed in 2016 and 2018. The rest of the research remains completely intact. Even in the case of the similarity equations with the Miller (1998, [13]), Dr. Proгри produced the computational body of work that Miller or any other researcher would have simply dreamed of creating. Dr. Proгри's novelty goes far beyond the ability to produced novel closed form expression or equations that contain Kampé de Fériet functions, Dr. Proгри's novelty has to do with the computational ability of the Kampé de Fériet functions that none before his have been able to clearly demonstrate.

Was the research that Dr. Proгри performed *innovative*?

Innovative research is a search for new mathematical techniques, methods, and computational tools. It is derived from developing and optimizing well-known methodologies, enabling the implementation of new and better solutions. *Innovative* research focuses on creating new *ideas*, analyzing problems, diagnosing them, and identifying their causes.

The creation and expansion of a MATLAB library for the fast and efficient computation of the Kampé de Fériet functions, generalized Bessel distribution function, and parabolic cylinder function distributions was only made possible by means of innovative research that was entirely focusses on creating new ideas and new computational tools that were not present before the publication of the nine journal papers published by Proгри (2016, [1])-Proгри (2021, [9]).

Having once and for all established the foundation that Dr. Proгри's body of research work is entirely *independent, original, unique, novel*, and *innovative* it does not mean that it may not have any similarity with someone's else's work. These similarities are okay if they arrived independently and without any prior knowledge in the same result.

Hence, after I completed the peer-review of a journal paper that I do not wish to reference because I do not wish to criticize the authors for not understanding the similarities and differences between Proгри (2016, [1])-Proгри (2019, [4]) and Miller (1989, [11])-Miller (2003, [15]) I felt compelled to detail those in this letter that focusses more on the content and the explanation of a few equations.

This letter is organized as follows: in Sect. 2 special remarks on the generalized Bessel distribution CDF of the first kind is detailed. Special remarks on the generalized Bessel distribution CDF of the second kind are touched on in Sect. 3. Section IV summarized special remarks on the GPCF distribution CDF; Conclusion is provided in Section V along with a list of references.

2 Special Remarks on the Generalized Bessel Distribution CDF of the First Kind

In 2016 I stated that Dr. Proгри was the first to have produced the closed form expression of the generalized Bessel distribution cdf of the first kind (see Proгри 2016, [1]). I believe that this statement needs some clarification in light of new evidence and information.

In 2016 I was not aware of the original work that Miller

(1998, [2]) had performed in obtaining an original closed form expression of the incomplete Lipschitz-Hankel integrals of Anger and Weber functions that can be employed to produce an expression of the generalized Bessel function cdf of the first kind. By way of this letter I would like to show the similarities and differences of Proгри (2016, [1] and 2018, [3]) and Miller (1998, [13]) work.

From (77) from Proгри (2016, [1])

$$F_{\text{GBessel1}}(x; a, d, p) = \frac{\int_0^x t^p e^{-td} I_p(t) dt}{C_1(p, d)} \quad (1)$$

Where $C_1(p, d)$ is given by (6) and (84) Proгри (2016)

$$C_1(p, d) = \frac{2^p \Gamma(p_1)}{\sqrt{\pi} |1-d^2|^{p_1}} = \frac{\Gamma(2p_1)}{2^p |1-d^2|^{p_1} \Gamma(p_2)} \quad (2)$$

All other parameters and variables are defined in Proгри (2016, [1]).

From (49) in Proгри (2018, [3]) we obtain:

$$F_{\text{GBessel1}}(x; a, d, p) = \frac{(d^2-1)^{p_1} x^{2p_1} \left(\frac{1}{p_1} K_1 - \frac{dx}{p_2} K_2 \right)}{2 \Gamma(2p_1)} \quad (3)$$

Where K_1 and K_2 are the Kampé de Fériet functions that are computed from Proгри (2018, [3])

$$K_1 = F_{1:1;1}^{1:0;0} \left[\begin{matrix} p_1: -; -; \\ p_3: p_2; \frac{1}{2}; x_1, x_2 \end{matrix} \right] \quad (4)$$

$$K_2 = F_{1:1;1}^{1:0;0} \left[\begin{matrix} p_2: -; -; \\ p_4: p_2; \frac{3}{2}; x_1, x_2 \end{matrix} \right] \quad (5)$$

And x_1 and x_2 are given by (116a) in Proгри (2016, [1]) or (22) in Proгри (2018, [3])

$$x_1 = \frac{x^2}{4} \quad (6)$$

$$x_2 = \frac{x^2 d^2}{4} \quad (7)$$

Hence, if we re-arrange (1) and substitute (3) the following maybe obtained

$$\begin{aligned} \int_0^x t^p e^{-td} I_p(t) dt &= F_{\text{GBessel1}}(x; a, d, p) C_1(p, d) \\ &= \frac{x^{2p_1}}{2^{p_2} \Gamma(p_2)} \left(\frac{1}{p_1} K_1 - \frac{dx}{p_2} K_2 \right) \end{aligned} \quad (8)$$

In Miller ((2.3) 1998, [13]), if we make the following substitutions

$$z = x \quad (9)$$

$$a = -d \quad (10)$$

$$\mu = 2p \quad (11)$$

$$r = 0 \quad (12)$$

$$s = 1 \quad (13)$$

$$(\xi_r) = - \quad (14)$$

$$(\eta_s) = p_2 \quad (15)$$

$$b = 1 \quad (16)$$

we have

$$\int_0^x \frac{t^p}{e^{td}} {}_0F_1[-; p_2; t_1] dt = \frac{x^{2p_1}}{2} \left(\frac{1}{p_1} K_{M1} - \frac{dx}{p_2} K_{M2} \right) \quad (17)$$

where K_{M1} and K_{M2} are the Kampé de Fériet functions that are computed from Miller (1998, [13])

$$K_{M1} = F_{1:1;1}^{1:0;0} \left[\begin{matrix} p_1: -; -; \\ p_3: p_2; \frac{1}{2}; x_1, x_2 \end{matrix} \right] \quad (18)$$

$$K_{M2} = F_{1:1;1}^{1:0;0} \left[\begin{matrix} p_2: -; -; \\ p_4: p_2; \frac{3}{2}; x_1, x_2 \end{matrix} \right] \quad (19)$$

On the other hand we have

$$\begin{aligned} \int_0^x \frac{t^{2p}}{e^{td}} {}_0F_1[-; p_3; t_1] dt &= \Gamma(p_2) \int_0^x e^{-td} t^{2p} \left(\frac{t}{2} \right)^{-p} I_p(t) dt \\ &= \Gamma(p_2) 2^p \int_0^x e^{-td} t^p I_p(t) dt \end{aligned} \quad (20)$$

From where we obtain that

$$\begin{aligned} \int_0^x e^{-td} t^p I_p(t) dt &= \frac{\int_0^x \frac{t^{2p}}{e^{td}} {}_0F_1[-; p_3; t_1] dt}{\Gamma(p_2) 2^p} \\ &= \frac{x^{2p_1}}{\Gamma(p_2) 2^{p_2}} \left(\frac{1}{p_1} K_{M1} - \frac{dx}{p_2} K_{M2} \right) \end{aligned} \quad (21)$$

Similarly, if we apply the same substitution in Miller ((2.4) 1998, [13]), we obtain:

$$\int_0^x \frac{t^p}{e^{td}} {}_0F_1[-; p_2; t_1] dt = \frac{x^{2p_1} e^{-dx}}{2} \left(\frac{1}{p_1} K_{M3} + \frac{dx}{p_2} K_{M4} \right) \quad (22)$$

where K_{M3} and K_{M4} are the Kampé de Fériet functions that are computed from Miller (1998, [13])

$$\begin{aligned} K_{M3} &= F_{2:1;0}^{0:2;1} \left[\begin{matrix} -; p_1, p_2; 1; \\ p_3, p_2; p_2; -; x_1, x_2 \end{matrix} \right] \\ &\equiv F_{2:0;0}^{0:1;1} \left[\begin{matrix} -; p_1; 1; \\ p_3, p_2; -; -; x_1, x_2 \end{matrix} \right]_{ii} \end{aligned} \quad (23)$$

$$\begin{aligned} K_{M4} &= F_{2:1;0}^{0:2;1} \left[\begin{matrix} -; p_1, p_2; 1; \\ p_4, p_3; p_2; -; x_1, x_2 \end{matrix} \right] \\ &= F_{2:0;0}^{0:1;1} \left[\begin{matrix} -; p_1; 1; \\ p_4, p_3; -; -; x_1, x_2 \end{matrix} \right] \end{aligned} \quad (24)$$

If we substitute (22) into (21) yields,

$$\int_0^x e^{-td} t^p I_p(t) dt = \frac{\int_0^x \frac{t^{2p}}{e^{td}} {}_0F_1[-; p_3; t_1] dt}{\Gamma(p_2) 2^p}$$

$$= \frac{x^{2p_1} e^{-dx}}{\Gamma(p_2) 2^{p_2}} \left(\frac{1}{p_1} K_{M_3} + \frac{dx}{p_2} K_{M_4} \right) \text{iii} \quad (25)$$

From Proгри (2018, (25) [3]) we obtain:

$$F_{\text{GBessel1}}(x; a, d, p) = \frac{(d^2 - 1)^{p_1} x^{2p_1} e^{-dx} \left(\frac{1}{p_1} K_3 + \frac{dx}{p_2} K_4 \right)}{2\Gamma(2p_1)} \quad (26)$$

where K_3 and K_4 are the Kampé de Fériet functions that are computed from Proгри (2018, [3])

$$K_3 \equiv F_{2;0;0}^{0;1;1} \left[\begin{matrix} -; p_1; 1; \\ p_3, p_2; -; -; x_1, x_2 \end{matrix} \right] \quad (27)$$

$$K_4 = F_{2;0;0}^{0;1;1} \left[\begin{matrix} -; p_1; 1; \\ p_4, p_3; -; -; x_1, x_2 \end{matrix} \right] \quad (28)$$

Similarly, if we re-arrange (1) and substitute (26) the following maybe obtained

$$\begin{aligned} \int_0^x t^p e^{-td} I_p(t) dt &= F_{\text{GBessel1}}(x; a, d, p) C_1(p, d) \\ &= \frac{x^{2p_1} e^{-dx}}{2^{p_2} \Gamma(p_2)} \left(\frac{1}{p_1} K_3 + \frac{dx}{p_2} K_4 \right) \end{aligned} \quad (29)$$

Since (8)/(29) is identical to (21)/(25), it is clear that (49)/(25) in Proгри (2018, [3]) or the closed form expression2 of the generalized Bessel cdf of the first kind are special cases of Millers (2.3) and (2.4) (1998, [13]) for the values of the parameters given in (9)-(16).

Yes, Dr. Proгри is the first to have derived the closed form expression of the generalized Bessel distribution cdf of the first kind from the direct integration; however, one can obtain the identical result from Miller (2.3) and (2.4) (1998, [13]) for the values of the parameters given in (9)-(16). And since this was done only after the fact that Dr. Proгри had already published his first journal paper Proгри (2016, [1]) it provides a bridge between two pioneers on the field of the confluent hypergeometric functions and Kampé de Fériet functions.

We conclude the discussion on the special remarks on the generalized Bessel distribution cdf of the first kind.

3 Special Remarks on the Generalized Bessel Distribution CDF of the Second Kind

In the previous section we were successful to show from Miller (2.3) and (2.4) (1998, [13]) one can derive the generalized Bessel distribution cdf of the first kind and produce exactly the same results as in Proгри (2018, (49) [3]).

Could either the Miller (2.3) and (2.4) (1998, [13]) have been used to derive the generalized Bessel distribution CDF of the

second kind given by Proгри (2016, [1]) and Proгри (2019, [4])?

The purpose of this section is to provide a definite answer to this question for two cases: the non-integer case and the integer case.

3.1 The Non-integer Case of the Generalized Bessel Distribution CDF of the Second Kind

The non-integer case of the generalized Bessel distribution CDF of the second kind is given by Proгри (2016, (142) [1]).

If we examine Proгри (2016, (126) [1])

$$\mathcal{F}_{\text{GBessel2}}(x; a, d, p) = \frac{\int_0^x t^p e^{-td} I_{-p}(t) dt - \int_0^x t^p e^{-td} I_p(t) dt}{\tilde{C}_2(p, d)} \quad (30)$$

we recognize two components: the first one of which is given by

$$\mathcal{F}_{\text{GBessel21}}(x; a, d, p) = \frac{\int_0^x t^p e^{-td} I_{-p}(t) dt}{\tilde{C}_2(p, d)} \quad (31)$$

And the second one of which is given by

$$\mathcal{F}_{\text{GBessel22}}(x; a, d, p) = \frac{-\int_0^x t^p e^{-td} I_p(t) dt}{\tilde{C}_2(p, d)} \quad (32)$$

For the second component we should be able to obtain an identical closed form expression using either Miller's (2.3) or (2.4) (1998, [13]) as in Proгри (2016, (128) [1]).

$$\int_0^x \frac{t^p I_p(t) dt}{e^{td}} = \frac{\left[\frac{F_{2;0;0}^{0;1;1} \left[\begin{matrix} -; p_1; 1; \\ p_3, p_2; -; -; x_1, x_2 \end{matrix} \right]}{C_{11}^{-1}(p, d)} + \frac{x F_{2;0;0}^{0;1;1} \left[\begin{matrix} -; p_1; 1; \\ p_4, p_3; -; -; x_1, x_2 \end{matrix} \right]}{C_{12}^{-1}(p, d)} \right] x^{2p_1}}{C_1^{-1}(p, d)} \quad (33)$$

However, if we were to employ either Miller's (2.3) and/or (2.4) (1998, [13]) to derive (31), that would have required a value of

$$\mu = -2p < -1; \text{ if } p > 1 \quad (34)$$

However, according to Miller's (2.3) and/or (2.4) (1998, [13]) we have the following restrictions for

$$\mu > -1 \quad (35)$$

The integral (31) that is given in Proгри (2016, (141) [1])

$$\int_0^x t^p e^{-td} I_{-p}(t) dt = \frac{2^p x e^{-dx}}{\Gamma(1-p)} \left[K_5 + \frac{dx}{2} K_6 \right] \quad (36)$$

Where K_5 and K_6 are the Kampé de Fériet functions that are computed from Proгри (2016, [1])

$$K_5 = F_{2;1;0}^{0;2;1} \left[\begin{matrix} -; 1, \frac{1}{2}; 1; \\ \frac{3}{2}, 1; 1-p; -; x_1, x_2 \end{matrix} \right] \quad (37)$$

$$K_6 = F_{2;1;0}^{0;2;1} \left[\begin{matrix} -; 1, \frac{1}{2}; 1; \\ 2, \frac{3}{2}; 1-p; -; x_1, x_2 \end{matrix} \right] \quad (38)$$

Since (31) was derived directly, that would have been impossible to have been derived using either the Miller's (2.3) and/or (2.4) (1998, [13]). Even if we were to relax the requirements in either (34) or (35) it would be impossible to come up with a set of values that would equate any of the Miller's equations (1998, [13]) to (37) or (38).

Is the non-integer case of the generalized Bessel distribution CDF of the second kind given by Proгри (2016, (142) [1]) an original expression?

Since (142) would not have been entirely derived by either the Miller's (2.3) and/or (2.4) (1998, [13]) then Proгри (2016, (142) [1]) is an original expression.

Even though it just happened by accident that Proгри's (2016, (118) [1]) could have been derived from either the Miller's (2.3) and/or (2.4) (1998, [13]), on the other hand Proгри's (2016, (142) [1]) would not have been derived from either Miller's (2.3) and/or (2.4) (1998, [13]).

In conclusions Proгри's derivations in (2016, [1]) and (2018, [3]) are *original*, *novel*, *innovative*, and *unique* and it just happened by accident that some of the derivations in either Proгри (2016, [1]) and (2018, [3]) could have been derived differently employing either Miller's (2.3) and/or (2.4) (1998, [13]).

3.2 The Integer Case of the Generalized Bessel Distribution CDF of the Second Kind

When I produced the integer case of the generalized Bessel function distribution of the second kind in Proгри (2019, [4]) I realized how difficult and computationally laborious this problem was but moreover how unlikely that someone else would have been able to have produced these formulae without the help of my MATLAB library.

As I examined Miller (1989, [11])-Miller (2003, [15]) I found absolutely no way how any of his formulae could, would or should have been used to derive any of my equations in Proгри (2019, [4]).

Moreover, the editorial office of the journal paper who asked me to review one of their journal papers has not been able to ask me to review another journal paper what may have any similarities with my work.

In addition, I found absolutely no other publications whether it be from Google Scholar or the Ithenticate License that I have obtained in 2021 to show once more that Proгри is (2019, [4]) *independent*, *original*, *unique*, *novel*, and *innovative*.

4 Special Remarks on the GPCF Distribution CDF

I took pride in the derivations of the journal papers on the GPCF distribution CDF in Proгри (2016, [5])-Proгри (2021, [9]). I produced two superb independent methodologies and four different means of successfully, accurately, and super efficient algorithms completely on my own.

If finding a similarity with my work in Proгри (2019, [4]) would have been unlikely, doing the same for my work in Proгри (2016, [5])-Proгри (2021, [9]) would have been highly unlikely.

I performed a google scholar work with the following settings: "Parabolic cylinder function distribution" with "empty" author field from 1980 until 2021 and only nine of twelve the total results that showed are my journal papers other two journal papers are the journal papers that I have reviewed, and another one which I have referenced early on in my research in Proгри (2016, [5]).

Beyond the Google Scholar and Ithenticate License it is not really my job neither to look nor examine any further research that may have or may not have any similarities with any of my publications in Proгри (2016, [5])-Proгри (2021, [9]).

5 Conclusions

In deriving the conclusions of this journal letter all the statement made in Proгри (2016, [1])-Proгри (2021, [9]) remain intact.

Four independent reviews and examinations of my research were conducted: self-review, Google Scholar search, Ithenticate Software License, and an independent publishing review office.

This letter summarizes the results from the extensive peer-review my research from 2016 until 2021 and points out that Dr. Proгри's research is *independent*, *original*, *unique*, *novel*, and *innovative*.

I want to conclude this research letter and thank everyone who was involved for providing valuable comments to my research.

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I want to profoundly thank the MathWorks at Natick, Massachusetts for providing a sponsored MATLAB licence [17] to Giftet Inc. as part of the Indoor Geolocation Systems MATLAB Library development that will enable the results of this work to be published in Dr. Progri pioneer publication *Indoor Geolocation Systems—Theory and Applications. Vol. I* (Not yet available in print) [16].

This journal paper is dedicated to four special men in my life: my grandfather, Xhevdet Progri, my dear father, Fiqiri Progri, my father's first cousin Dr. Peter Demir, and Qazim Demir, the brother of my grandfather, Xhevdet Progri.

This journal paper is also dedicated to the Golden Bear, Jack Nicklaus, the greatest golfer of all time who won six Masters Championships the most of all time. Needless, to say I have fallen in love with his masterpiece book, *Golf My Way*. Moreover, Jack Nicklaus [18] reminds me of my grandfather who I loved him very much.

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ⁱ Who is Allen R. Miller, the author of references [11]-[15]? A brilliant mathematician who worked in the Nava Research Laboratory, Washington, DC between 1989 to 2003 and in addition to the contributions mentioned in [10] also made signification contributions in understanding and reducing the Kampé de Fériet functions by means of the confluent hypergeometric functions and other elementary functions.

ⁱⁱ There is a typo in Miller (1998 (2.4), [13]) the coefficient in the first Kampé de Fériet function that is $\frac{2+\mu}{3}$ should be $\frac{2+\mu}{2}$.

ⁱⁱⁱ I should indicate that Miller was not aware that his expressions would have been used to derive the general Bessel function distribution of the first kind, I was not aware of the Miller's expressions when I derived the Bessel function distribution of the first kind, even when I conducted the review of a journal paper in October of 2018 through March of 2019 it was me who showed to the authors of that journal paper what the similarities of the Miller and Progre were. Nevertheless, I felt compelled that I need to do more than just conduct the review of that journal paper; hence, I decided to write this letter.