

Technical Report

A Navigation Algorithm Using a System of GNSS Satellites and Stationary Pseudolites

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Abstract—The purpose of this work¹ is to assess the accuracy of different navigation algorithms utilized for a combined system of GPS satellites and pseudolites. The algorithms under investigation are the ordinary pseudorange least square algorithm (LSA), single difference and double difference Kalman based algorithms, and a derived algorithm which employs the Doppler measurements. An integrity measure of the combined system for the different algorithms is also provided along with the navigation accuracy. A list of pros and cons for different algorithms follows the last section of the paper.

Index Terms—Pseudorange, Doppler, accumulated carrier phase, measurements, navigation, pseudolites, Kalman, least square algorithm, ionospheric propagation delay, indoor terrestrial applications.

1 Introduction

The design and implementation of a robust, highly accurate filter has been the focus of our research in recent years. Initially, we attempted to design a filter, which would perform a CAT III precision landing utilizing only a system of pseudolites [1]-[4].

The filter design was based upon the technique of double difference and thus required a minimum of two receivers and two satellites in view in order to have one independent measurement all the time. The evaluation of the probability of false alarm and miss-detection was an important part of the

system design and its overall performance. Although the navigation performance was exceeding the FAA requirements for CAT III landing and the integrity performance was close to the corresponding FAA requirements, the complexity involved with processing data from two separated receivers is usually undesirable. In order to improve the system integrity and assure system operation even under severe geometry, weather, and interference environments we considered a combined system of satellites and pseudolites [4]-[6].

In our research of the indoor geo-location applications, we utilized a Kalman based filter to process indoor geo-location data [7], [8].

This work constitutes our base line or serves as the corner stone. In an attempt to reduce the system complexity, introduce something novel, and at the same time maintain the system accuracy, we resurrected the Doppler based navigation idea [9] which is being presented in [10], [11].

This paper, thus serves as the synthesis of our research to obtain an easily implemental, highly accurate filter when a single receiver processes all measurements on the fly.

This paper is organized as follows: In Sect. 2 discusses all the navigation algorithms description is researched and implemented. Next, we proceed with the integrity measurement in Sect. 3. The simulation of modified LSA, single-double difference Kalman filter, and Derived Doppler is provided in Sect. 4. Conclusion is summarized in Sect. 5 followed by acknowledgment in Sect. 6 and references in Sect. 7.

2 Navigation Algorithm Description

Although most of the navigation algorithms are discussed previously [1]-[11], the intent here is to consider their design from the point of view of a system with combined satellites and pseudolites. We have divided navigation algorithms into three categories: LSA pseudorange based, accumulated carrier Kalman based, and derived Doppler LSA based.

The algorithm description section contains the ordinary pseudorange least square, Single Difference and Double Difference, and Doppler Derived Least Squares.

2.1 Ordinary Pseudorange Least Square

Consider a system of combined pseudolites and satellites as shown in Fig. 1 and Combined SAT/PSL raw pseudorange filter design is depicted in Fig. 2.

For the reasons explained in [7], [8], we will no longer deal with the snap shot solution (except when the distances between the user and the source are within one sigma value of the measurement noise); therefore, consider the incremental solution formulation for the k^{th} epoch and for either the satellites (S) or pseudolites (P) as,

$$\Delta \hat{\mathbf{s}}_i^k = \left(\mathbf{H}_i^{kT} \mathbf{R}_i^{k-1} \mathbf{H}_i^k \right)^{-1} \mathbf{H}_i^{kT} \mathbf{R}_i^{k-1} \mathbf{r}_i^k, \quad i = \{S, P\} \quad (1)$$

Where

$\Delta \hat{\mathbf{s}}_i^k$ is the increment in the state vector,

$\mathbf{H}_i^k$ is the matrix which relates the residual vector with the state vector,

$\mathbf{R}_i^k$ is the pseudorange measurement noise covariance matrix,

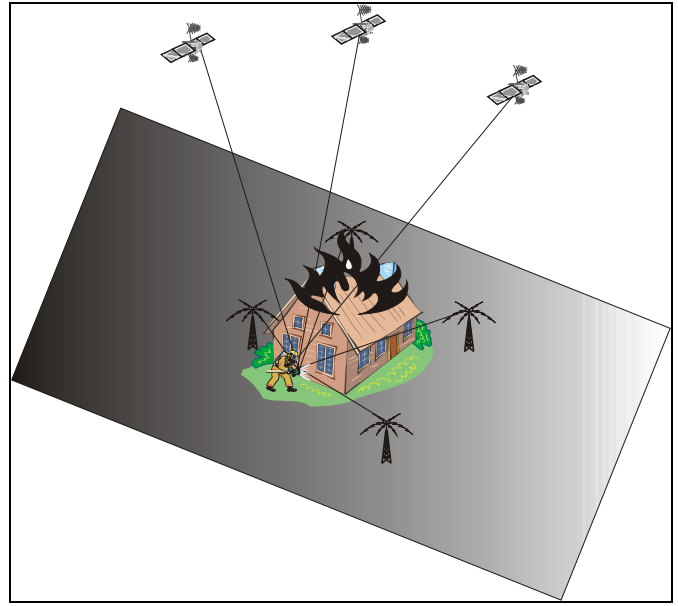


Fig. 1. Combined SAT/PSL system without reference station.

$\mathbf{r}_i^k$ is the residual vector determined from

$$\mathbf{r}_i^k = \boldsymbol{\rho}_i^k - \hat{\mathbf{d}}_i^k(\hat{\mathbf{s}}_i^{k-1}), \quad i = \{S, P\} \quad (2)$$

In (2), $\boldsymbol{\rho}_i^k$ denotes the raw pseudorange measurement vector and $\hat{\mathbf{d}}_i^k$ is the range estimate as a function of the state vector, $\hat{\mathbf{s}}_i^k$.

We have assumed that the receiver is capable of tracking both pseudolite and satellite signals.

If we assume that in the beginning the filter processes satellite data then the solution for the state vector is of the form,

$$\hat{\mathbf{s}}_S^1 = \hat{\mathbf{s}}_0^0 + \Delta \hat{\mathbf{s}}_S^1 \quad (3)$$

where $\hat{\mathbf{s}}_0^0$ is the starting state vector.

Next, the filter (see Fig. 2) will process pseudolite data; therefore, the state solution is given by,

$$\hat{\mathbf{s}}_P^1 = \hat{\mathbf{s}}_S^1 + \Delta \hat{\mathbf{s}}_P^1 \quad (4)$$

The solution (4) will serve as the starting point for the satellite solution. Based on this argument, the solution for the state vector at the k^{th} epoch can be written as,

$$\hat{\mathbf{s}}_S^k = \hat{\mathbf{s}}_P^{k-1} + \Delta \hat{\mathbf{s}}_S^k \quad (5)$$

and state solution vector for processing pseudolite data,

$$\hat{\mathbf{s}}_P^k = \hat{\mathbf{s}}_S^k + \Delta \hat{\mathbf{s}}_P^k \quad (6)$$

The following diagram illustrates the sequence of the measurement processing and the state vector updates. One can easily reconstruct the combined PSL/SAT filter (see Fig. 2), if the filter state is updated first based on pseudolite data.

2.2 Single Difference and Double Difference

Point positioning and kinematic positioning can be

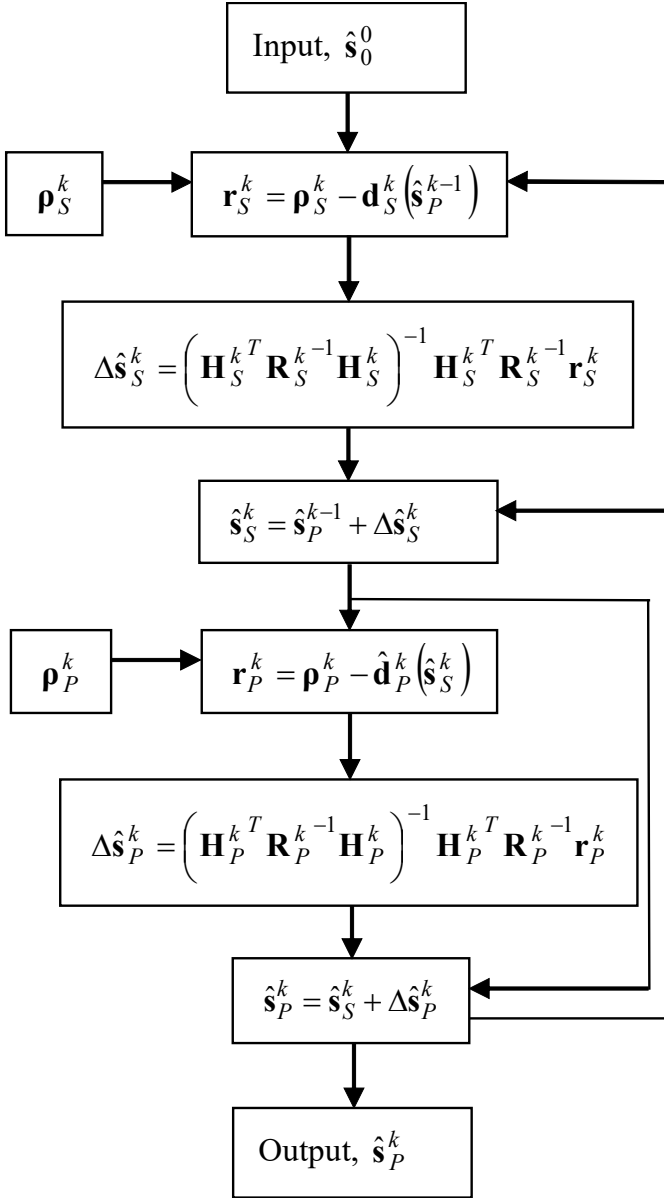


Fig. 2. Combined SAT/PSL raw pseudorange filter design.

accomplished using carrier phase and the technique of single or double differencing. Whichever the technique applied, the satellite with the highest elevation and the pseudolite with the lowest elevation are selected as the base satellite and pseudolite respectively. A detailed description of the system is given in Fig. 3.

A Kalman based filter utilizing single difference accumulated phase measurement was derived in [7]. The same filter will be generalized for a system of combined SAT/PSL data. Thus, based on equation (7) of [7], the single difference accumulated carrier phase vector can be written as,

$$\bar{\boldsymbol{\phi}}_i^k = \bar{\mathbf{d}}_i^k + \lambda \bar{\mathbf{a}}_i^0 + 2\bar{\mathbf{e}}_i^k, \quad i = \{S, P\} \quad (7)$$

Here, $\bar{\boldsymbol{\phi}}_i^k$ is the single difference accumulated phase vector, $\bar{\mathbf{d}}_i^k$ is the estimated single difference range, $\lambda \bar{\mathbf{a}}_i^0$ denotes the modified single difference ambiguities (see (4) of [7]), $\bar{\mathbf{e}}_i^k$ is

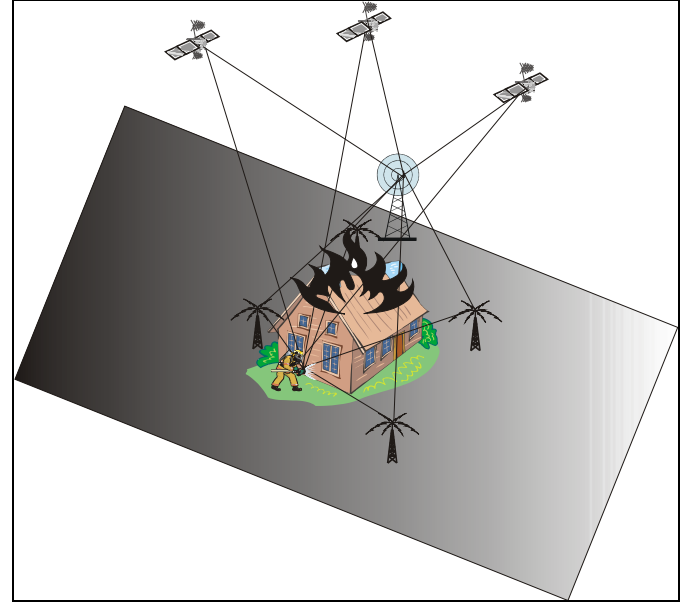


Fig. 3. Combined SAT/PSL system with reference station.

the error term, and λ of course denotes the wavelength of the carrier.

Similar to the single difference equation, one can formulate the double difference accumulated carrier phase measurement vector, $\bar{\boldsymbol{\phi}}_i^k$, as [8],

$$\bar{\boldsymbol{\phi}}_i^k = \bar{\mathbf{d}}_i^k + \lambda \bar{\mathbf{a}}_i^0 + 2\bar{\mathbf{e}}_i^k, \quad i = \{S, P\} \quad (8)$$

Under the assumption that $\boldsymbol{\phi}_i^k$ can be either the single difference, $\bar{\boldsymbol{\phi}}_i^k$, or the double accumulated phase, $\bar{\boldsymbol{\phi}}_i^k$, the diagram of the Kalman based filter is pictured in Fig. 4. More details of the double difference technique can be found in [13].

2.3 Doppler Derived Least Squares

Assuming an initial condition for the state vector $\hat{\mathbf{s}}_0^0$, the Doppler derived pseudorange can be formed as [10], [11],

$$\hat{\rho}_i^k = \hat{\mathbf{d}}_i^{k-1} + \dot{\boldsymbol{\phi}}_i^{k-1} + 0.5(\dot{\boldsymbol{\phi}}_i^{k-1} - \dot{\boldsymbol{\phi}}_i^{k-2}) + \mathbf{w}_i^{k-1} \quad (9)$$

In the above expression, the explanation of the unknowns reads:

$\hat{\rho}_i^k$ is the Doppler derived pseudorange;

$\hat{\mathbf{d}}_i^{k-1}$ is the estimated source-user (or transmitter-receiver) range;

$\dot{\boldsymbol{\phi}}_i^{k-1}$ is the Doppler between the user (or receiver) and source;

$\mathbf{w}_i^{k-1}$ denotes the process noise;

subscript i is the index for satellite (S) or pseudolite (P) respectively;

superscript k is the epoch index.

Although in [10] we did not include the second derivative of the user-source range, we have included that here assuming that

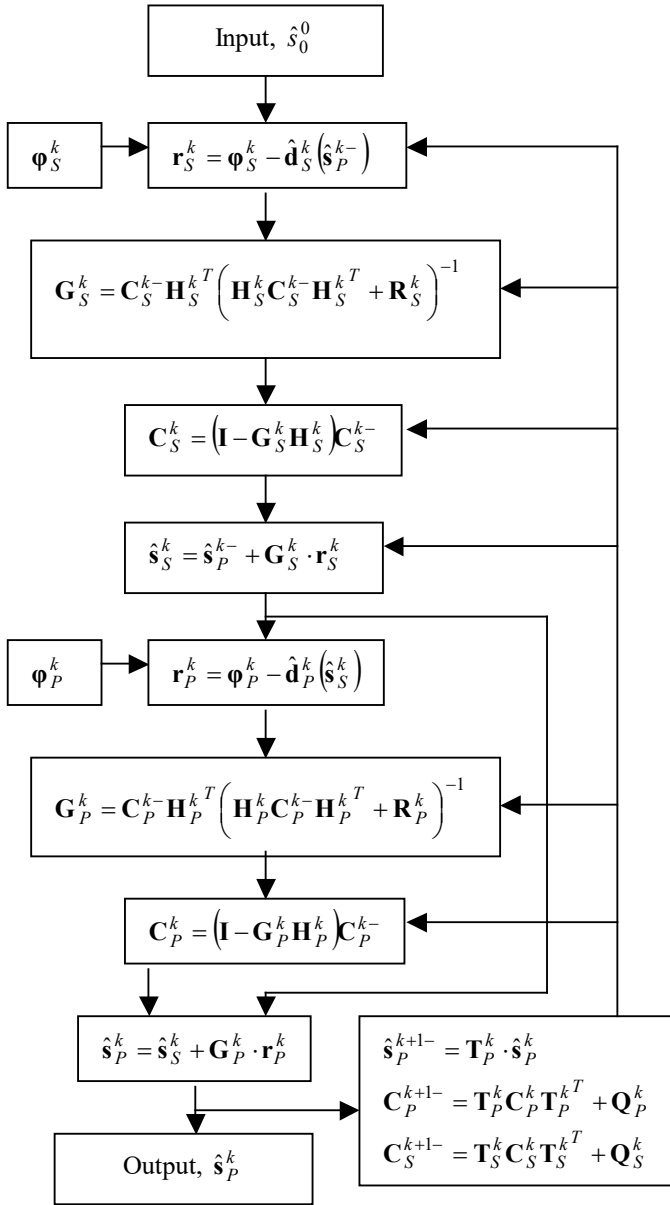


Fig. 4. Combined SAT/PSL single/double difference filter.

the trajectory includes acceleration or higher order derivatives.

At this point we should be able to modify the filter diagram 1 to reflect relation (7) (see Fig. 5). While constructing the filter, which processes the SAT and PSL data, we have assumed that the filter processes first the SAT data and then the PSL data.

3 Integrity Assessment

Integrity is defined as the measure of trust that can be placed on either a GNSS signal that is being acquired, tracked, processed, and/or the navigation solution [12]. Broadly speaking integrity consists of two main components: *signal integrity*ⁱⁱ (See Parkinson et al. 1995 [9], pg. 128, 158, 160) and *navigation system algorithm or performance integrity* [4], [6], [8], (See Parkinson et al. 1995 [9], pg. 151, 160), [12], [14].

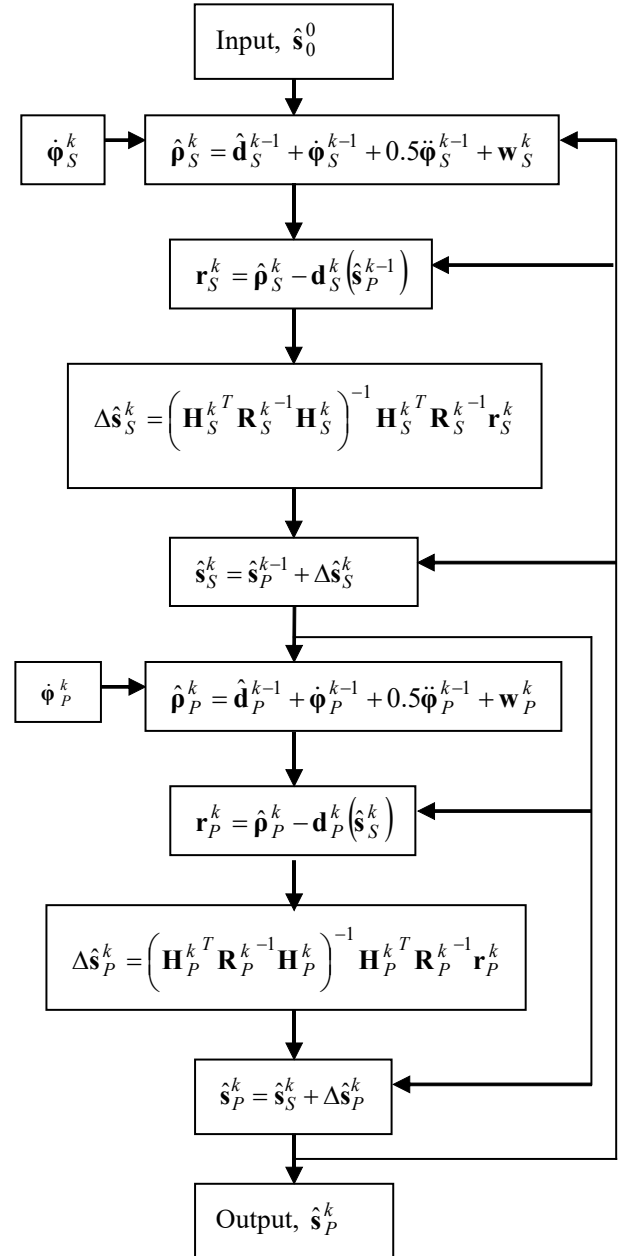


Fig. 5. Combined SAT/PSL incremental Doppler derived filter.

In order to provide a means for [navigation system algorithm] integrity we will consider the residual formulation,

$$\mathbf{r}_i^k = \mathbf{m}_i^k - \hat{\mathbf{m}}_i^k(\hat{\mathbf{s}}_i^{k-1}), i = \{S, P\} \quad (10)$$

where, $\mathbf{m}_i^k$ is the measurement vector and $\hat{\mathbf{m}}_i^k(\hat{\mathbf{s}}_i^{k-1})$ is the nonlinear estimate of the measurement vector $\mathbf{m}_i^k$.

Once the incremental solution update is obtained from

$$\hat{\mathbf{s}}_i^k = \hat{\mathbf{s}}_i^{k-1} + \Delta \hat{\mathbf{s}}_i^k \quad (11)$$

$$\Delta \hat{\mathbf{s}}_i^k = (\mathbf{H}_i^{kT} \mathbf{R}_i^{k-1} \mathbf{H}_i^k)^{-1} \mathbf{H}_i^{kT} \mathbf{R}_i^{k-1} \mathbf{r}_i^k \quad (12)$$

the residual vector is recomputed from,

$$\mathbf{r}_i^{k+} = \mathbf{m}_i^k - [\hat{\mathbf{m}}_i^k(\hat{\mathbf{s}}_i^k) \equiv \hat{\mathbf{m}}_i^k(\hat{\mathbf{s}}_i^k + \Delta \mathbf{e}_{si}^k)] = \mathbf{r}_i^k + \Delta \mathbf{e}_{ri}^k \quad (13)$$

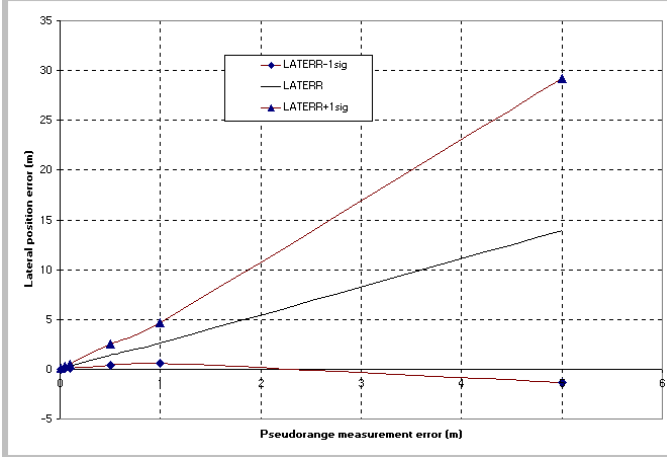


Fig. 6. LPE using modified pseudorange LSA.

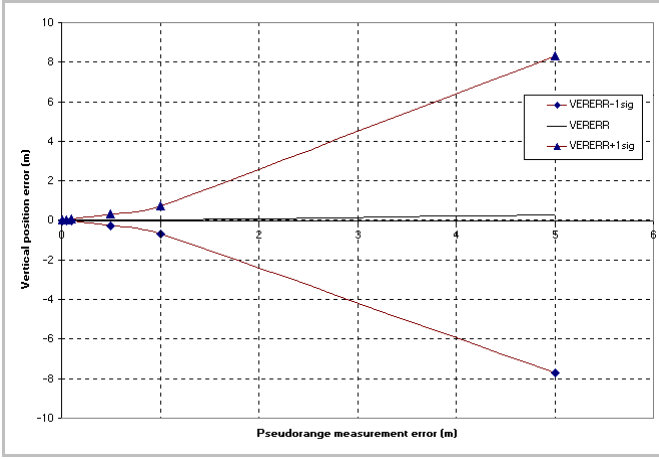


Fig. 7. VPE using modified pseudorange LSA.

The nonlinear relation between the error in the residual vector, $\Delta \mathbf{e}_{ri}^k$, and the solution increment, $\Delta \mathbf{e}_{si}^k$, is given by the following set of measures,

3.1 Probability of False Alarm

It is the probability that the normalized second norm of the residual error is greater than the residual threshold under normal conditions [14]

$$P_{FA} = P \left\{ \frac{\|\Delta \mathbf{e}_{ri}^k\|}{N_r} > \tau_r | NC \right\} \quad (14)$$

where N_r is the size of the residual vector, τ_r is the threshold of the residual vector, and NC denotes normal condition.

3.2 Probability of Misdetection

It is the probability that the normalized norm of the residual threshold is smaller than the residual threshold at the same time the solution error is greater than the accepted solution accuracy [14], as

$$P_{MD} = P \left\{ \frac{\|\Delta \mathbf{e}_{ri}^k\|}{N_r} < \tau_r \cap \frac{\|\Delta \mathbf{e}_{si}^k\|}{N_s} < \tau_s \right\} \quad (15)$$

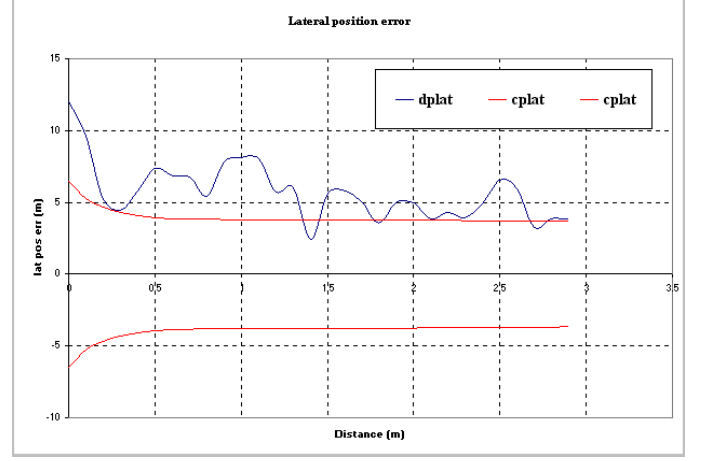


Fig. 8. LPE vs. distance using pseudorange.

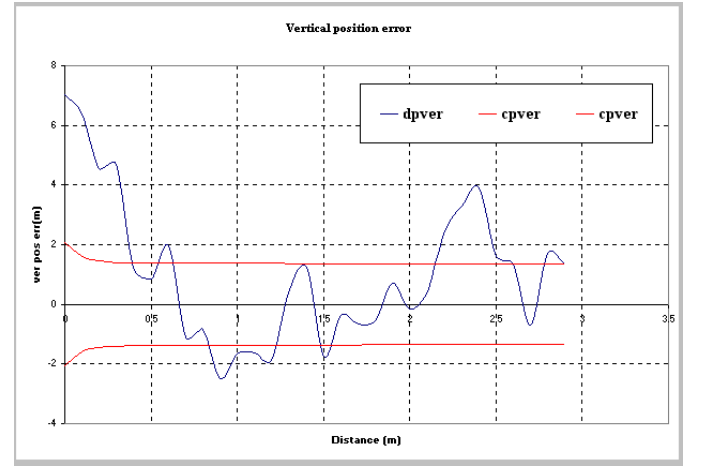


Fig. 9. VPE vs. distance using pseudorange.

where N_s is the size of the sub state vector. τ_s is the solution accuracy threshold.

4 Simulation

The simulation scenario is the same with the one corresponding to the moving firefighter in [10], [11].

4.1 Modified Pseudorange LSA

In Figs. 6 and 7 we present the lateral and vertical position error (LPE and VPE) vs. pseudorange measurement noise for the modified LSA algorithm.

4.2 Single-Double Difference Kalman Based

Kalman filter can be driven using either one of the measurements (pseudorange, accumulated carrier phase, or Doppler) or their combination.

When Kalman filter is driven with pseudorange only measurements, the lateral and vertical position and velocity

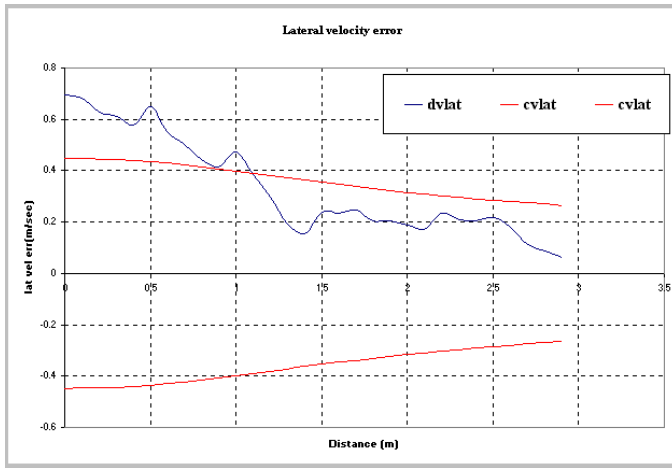


Fig. 10. LVE vs. distance using pseudorange.

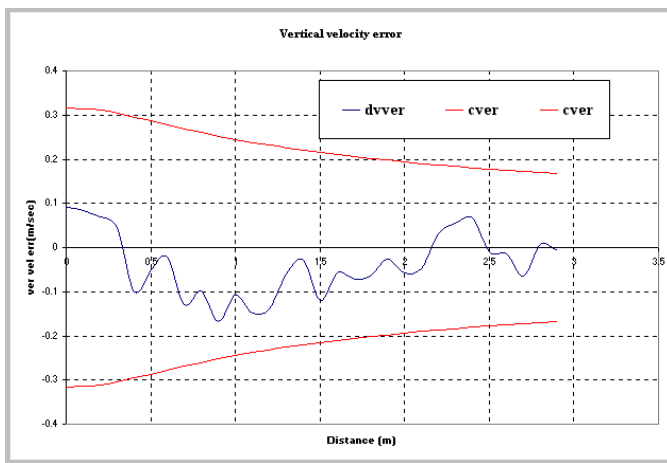


Fig. 11. VVE vs. distance using pseudorange.

errors (LPE, VPE, LVE, and VVE) vs. distance are pictured in Figs. 8 through 11.

When Kalman filter is driven with pseudorange and carrier phase measurements, the lateral and vertical position and velocity errors vs. distance are pictured in Figs. 12 through 15.

4.3 Derived Doppler

Similar to the pseudorange only case, we process Doppler measurements using the modified LSA.

The lateral and vertical position errors vs. Doppler measurement error are shown in Figs. 16 and 17.

5 Conclusions

For the system of combined pseudolites and satellites, we present the filter design assessment according to the accuracy (or navigation error mean and standard deviation in parentheses) in Tab. I.

According to the results presented in Tab. I, it appears that

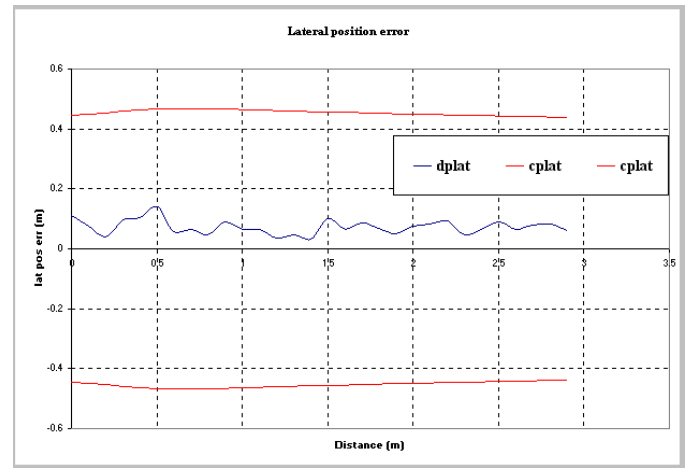


Fig. 12. LPE vs. distance using pseudorange and carrier phase.

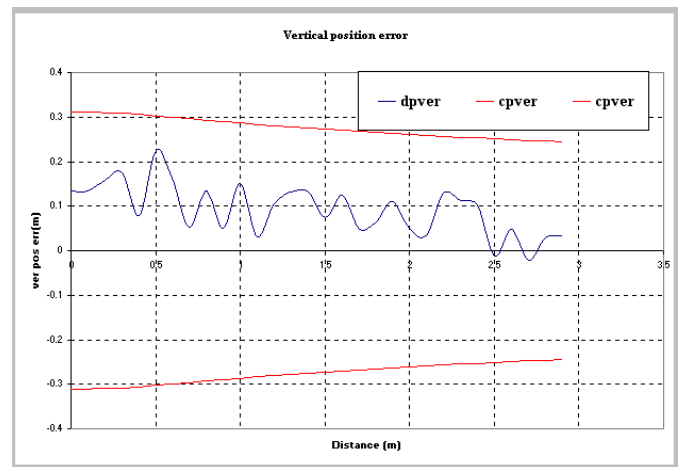


Fig. 13. VPE vs. distance using pseudorange and carrier phase.

the Kalman engine performs better than either one of the MLSAs.

If we split the cost of the filter in the cost of designing (CD) and the cost of tuning (CT), then the Kalman filter cost is higher than the MLSA as shown in Tab. II.

For distances comparable to 1 sigma value, the CLSA algorithm provides a better accuracy than the MLSA algorithm.

6 Acknowledgement

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I want to profoundly thank my Ph.D. advisor Prof. William R. Michelson for his support for my Ph.D. dissertation on [16] which served as the foundation for my book on *Indoor Geolocation Systems—Theory and Applications. Vol. I* (Not yet available in print) [17].

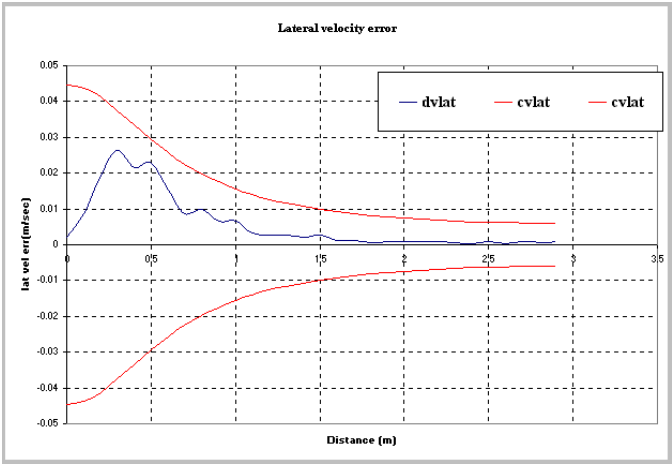


Fig. 14. LVE vs. distance using pseudorange and carrier phase.

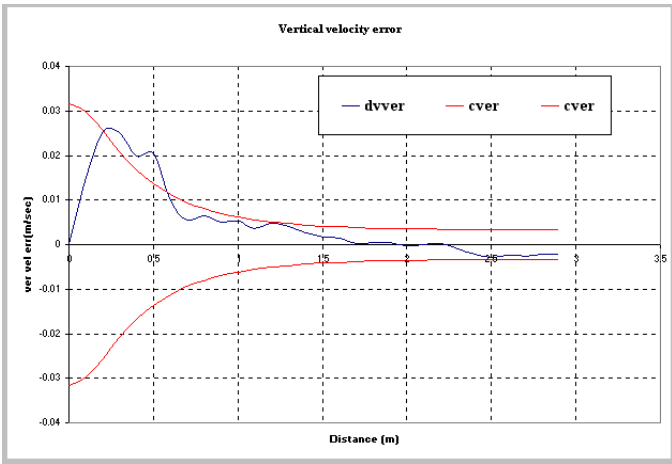


Fig. 15. VVE vs. distance using pseudorange and carrier phase.

TABLE I: METHODS' ASSESSMENT ACCORDING TO ACCURACY

Method	MLSA (PR)	Kalman (PR)	Kalman (PR&CP)	MLSA (DO)
LPE (m)	12 (15)	4 (4)	0.1 (0.4)	1.2 (0.5)
VPE (m)	0 (5)	1 (1.5)	0.1 (0.25)	0.25 (0.1)
LVE (m/s)	N/A	0.1 (0.3)	0.0 (0.00)	N/A
VVE (m/s)	N/A	0.01 (0.3)	0.0 (0.00)	N/A

7 References

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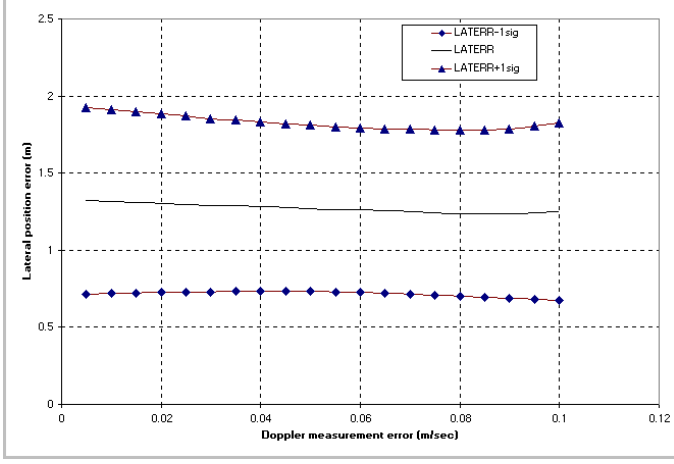


Fig. 16. LPE using Doppler derived pseudorange LSA.

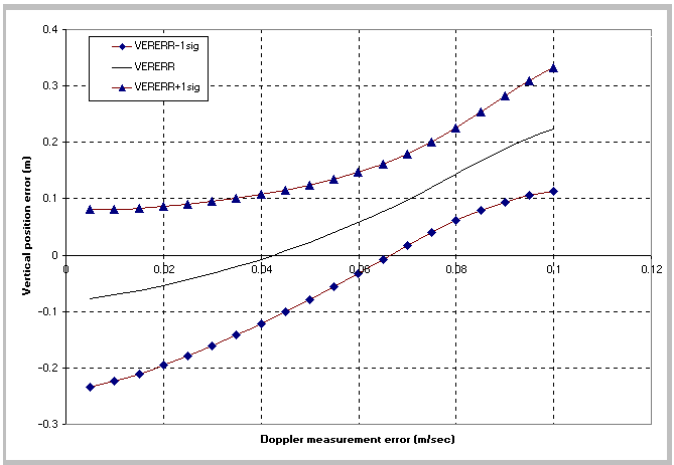


Fig. 17. VVE vs. distance using pseudorange and carrier phase.

TABLE II: METHODS' ACCURACY ACCORDING TO COST

Method	MLSA (PR)	Kalman (PR)	Kalman (PR&CP)	MLSA (DO)
Cost	CD	CD+CT	CD+CT	CD

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ⁱ *Editor's Note:* This article was originally published in 2001 [1]. The main results of this article are valid if only the data are assumed to be normally or Gaussian distributed. For other types of data distributions, I believe that a modification of the algorithm may be needed. The original publication contained a few typos and some minor language problems that I believe are fixed in this edition. A number of other clarifications are made in this publication and also the equation editor is greatly improved. Since, the original

article did not contain a DOI and it is a copyright by the authors I believe that the publication of this article in the form of a technical report is a good way to preserve the originality and authenticity of the article and provide a much.

ⁱⁱ In my earlier part of my research, I was mainly focused on the latter which is the *navigation system algorithm or performance integrity*. *Signal integrity* is much more difficult and it is not the subject of this paper, but it is discussed in my Ph.D. dissertation [17] and in my book [18].