

Technical Report

A GNSS Doppler Navigation Algorithm

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Abstract—This algorithm¹ uses the Doppler measurements that are produced from a GPS any other Global Navigation Satellite System (GNSS) receiver by acquiring and tracking GPS (or GLONASS or Galileo or any other GNSS) satellite signals for navigation. It is an innovative algorithm, because it employs only the Doppler measurements and because it requires a known initial location of the receiver. Initially, this algorithm was developed for stationary GPS (or GNSS) transmitters. In the article we have successfully attempted to adapt this algorithm for moving GNSS satellites. This algorithm appears to perform far better than the pseudorange conventional least square algorithm (LSA), due to the small standard deviation of the Doppler measurement noise. While we have already developed and tested this algorithm with simulated data that produces *submeter lateral/vertical position and submeter per second lateral/vertical velocity errors*, it still remains to test results with live GNSS satellites data or measurements.

Index Terms—Pseudorange, Doppler, accumulated carrier phase, measurements, navigation, pseudolites, Kalman, least square algorithm, ionospheric propagation delay, indoor terrestrial applications.

1 Introduction

The Doppler based navigation idea for GNSS applications is not new. In the late 50s Guier and Weiffenbach proposed a Doppler Satellite Navigation system, which could provide navigation accuracy of about *one-half mile* provided that proper use of the full Doppler measurement is made [2]. At that time both the navigation scientists and engineers were faced with challenge of designing an end-to-end system, which as we know is one of the predecessors of the GPS system. About the same time Guier and Weiffenbach proposed a technique for

measuring the Doppler shift of radio transmissions from satellites, thus enabling navigation based on their algorithm [2], [3]. Applications of Doppler measurements were also found in relativity, space probe tracking, and geodesy [4].

In the late 1960s, R.R. Newton, trying to persuade the space community, argued that it is feasible to build a system of navigation by measuring the Doppler shift in the radio transmissions from a near-Earth satellite and a simple equipment for conducting the measurements [5].

By considering a particular case, R.R. Newton, described in detail that the calculations required to produce a navigation solution accurate to about *500 m*, once the measurements are

recorded, can be performed even by hand within 10-15 min [5].

In the early 1970s J. Glish, proposed a closed form solution for a Doppler satellite navigation system [6], based on the early work proposed by R.R. Newton [5]; however, J. Glish does not present the expected accuracy of his approach.

The Doppler measurements were also combined with the range measurements to yield an instantaneous positioning with a single satellite [7]. This same idea is used by N. Levanon to produce user terminal position instantly by a two-communication system between a user terminal on the earth surface and a single low earth orbit (LEO) with reported along and cross track position and velocity standard deviation errors on the order on 30 m and 3m/s respectively [8].

Since the Doppler measurements are less noisy, an algorithm, which would produce the user's position based only on Doppler, would be very desirable. In [9] we have proposed an alternative algorithm that uses the Doppler measurements for stationary pseudolites that produces *submeter lateral/vertical position and submeter per second lateral/vertical velocity errors*.

Since, this approach appears feasible for stationary pseudolites we decided to modify this algorithm for moving satellites, which is explained in greater details in the following section.

First, we explore the idea of navigating with Doppler in a noiseless environment, which enables the derivations of all the main equations; and second, it is straightforward to adapt our approach into a noisy media.

For simulation purposes, we have investigated the stationary and moving user scenarios. The error on the Doppler measurement was estimated from recorded Doppler measurements at the Satellite Navigation Lab at WPI in 2000-2001 [9]-[11].

This paper is organized as follows: In Sect. 2 the algorithm description is given. Simulation of moving receiver is provided in Sect. 3. Conclusion is summarized in Sect. 4 followed by acknowledgment in Sect. 5 and a list of useful references in Sect. 6.

2 Doppler GNSS Algorithm Description

The Doppler GNSS algorithm description section contains the one-dimensional (1-D), two-dimensional (2-D), three-dimensional (3-D), and multi-D GNSS applications.

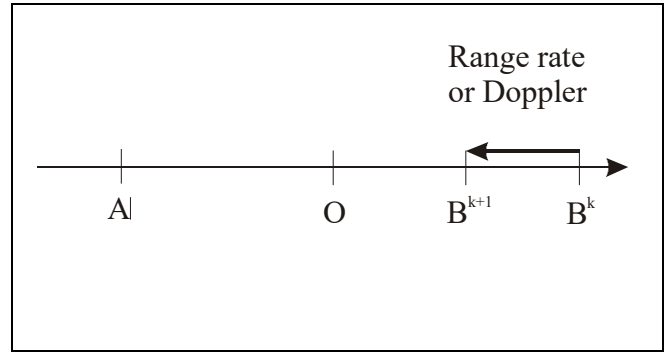


Fig. 1. 1-D GNSS system description.

2.1 One Dimensional GNSS Applications

Assume that one moving transmitter A (possible a GNSS satellite) and one receiver B are positioned on a 1-D axis as pictured in Fig. 1.

We further assume that the receiver B is perfectly synchronized with the transmitter A and that the initial location of the receiver B is known. As in [9], let d_{AB}^k denote the distance between the moving transmitter A and the moving receiver B at discrete time k . Assume that the receiver B moves towards the transmitter A by the amount of $\bar{v}_{AB}^k$. This is analytically determined from,

$$\bar{v}_{AB}^k \cong d_{AB}^{k+1} - d_{AB}^k \quad (1)$$

Assuming that the observations are performed at 1 Hz rate, relationship (1) gives the formula for approximating the average velocity during 1 sec interval. Under the above assumptions we can rewrite (1) is accordance with,

$$\bar{v}_{AB}^k \cong x_B^{k+1} - x_B^k - (x_A^{k+1} - x_A^k) \quad (2)$$

This expression enables us to form the navigation equation for the 1-D GNSS application, which can be written as,

$$x_B^{k+1} \cong x_B^k + \bar{v}_{AB}^k + x_A^{k+1} - x_A^k \quad (3)$$

It is evident in this case that by knowing the Doppler at any epoch we can determine the location of the moving receiver B with perfect accuracy in the 1-D GNSS applications.

Note: The same expression can be obtained when the receiver B moves away from transmitter A with the only modification of (3): the plus sign, (+), in front of $\bar{v}_{AB}^k$ now becomes minus, (-).

2.2 Two-Dimensional GNSS Applications

Assuming an error free environment with perfect synchronization between the transmitters and the receiver, at least two moving transmitters A and B are required to determine the correct location of a single receiver C in a two-dimensional

not in the same line at any time as shown in Fig. 3. Denote that the new ranges between receiver D and transmitters A, B and C are d_{AD}^{k+1} , d_{BD}^{k+1} , and d_{CD}^{k+1} respectively (see Fig. 3). Similar to the one-dimensional and two-dimensional cases the receiver D moves towards transmitters A, B, and C by the amount of,

$$\bar{v}_{AD}^k \cong d_{AD}^{k+1} - d_{AD}^k \quad (21)$$

$$\bar{v}_{BD}^k \cong d_{BD}^{k+1} - d_{BD}^k \quad (22)$$

$$\bar{v}_{CD}^k \cong d_{CD}^{k+1} - d_{CD}^k \quad (23)$$

The new point D^{k+1} is found from the intersection of three spherical surfaces—one with its center at transmitter A^{k+1} and with radius given by (21), one circle with its center at the transmitter B^{k+1} and with radius given by (22), and the final circle with its center at transmitter C^{k+1} and with radius given by (23),

$$d_{AD}^{k+1} \cong d_{AD}^k + \bar{v}_{AD}^k \quad (24)$$

$$d_{BD}^{k+1} \cong d_{BD}^k + \bar{v}_{BD}^k \quad (25)$$

$$d_{CD}^{k+1} \cong d_{CD}^k + \bar{v}_{CD}^k \quad (26)$$

Without showing all the work the solution for the new location of the receiver D is determined from:

$$x_D^{k+1} = d_1 z_D^{k+1} + e_1 \quad (27)$$

$$y_D^{k+1} = d_2 z_D^{k+1} + e_2 \quad (28)$$

$$z_D^{k+1} = \frac{-b_3 \pm \sqrt{b_3^2 - a_3 c_3}}{a_3} \quad (29)$$

Where the unknown parameters are determined from

$$a_3 = d_1^2 + d_2^2 + 1 \quad (30)$$

$$b_3 = d_1 e_1 + d_2 e_2 - d_1 x_C^{k+1} - d_2 y_C^{k+1} - z_C^{k+1} \quad (31)$$

$$c_3 = e_1^2 + e_2^2 - 2e_1 x_C^{k+1} - 2e_2 y_C^{k+1} - A_3 \quad (32)$$

$$d_1 = 2 \frac{b_1(z_C^{k+1} - z_B^{k+1}) - b_2(z_C^{k+1} - z_A^{k+1})}{a_1 b_2 - a_2 b_1} \quad (33)$$

$$d_2 = 2 \frac{a_2(z_C^{k+1} - z_A^{k+1}) - a_1(z_C^{k+1} - z_B^{k+1})}{a_1 b_2 - a_2 b_1} \quad (34)$$

$$a_1 = 2(x_C^{k+1} - x_A^{k+1}) \quad (35)$$

$$a_2 = 2(x_C^{k+1} - x_B^{k+1}) \quad (36)$$

$$b_1 = 2(y_C^{k+1} - y_A^{k+1}) \quad (37)$$

$$b_2 = 2(y_C^{k+1} - y_B^{k+1}) \quad (38)$$

$$c_1 = A_1 - 2z_D^{k+1}(z_C^{k+1} - z_A^{k+1}) \quad (39)$$

$$c_2 = A_2 - 2z_D^{k+1}(z_C^{k+1} - z_B^{k+1}) \quad (40)$$

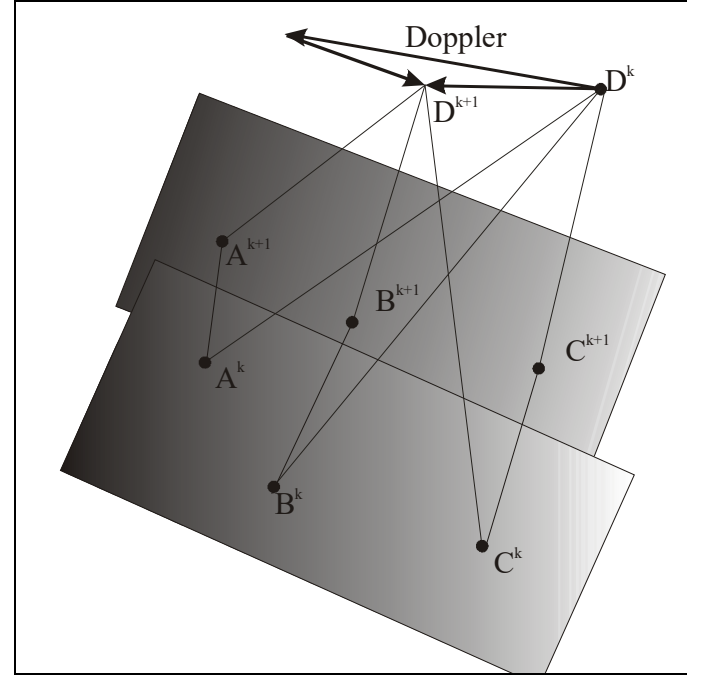


Fig. 3. 3-D system description.

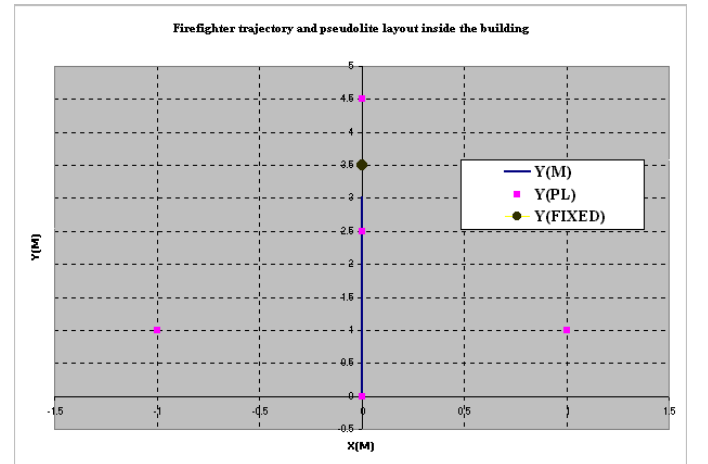


Fig. 4. GNSS satellite, moving receiver system description.

$$e_1 = \frac{b_2 A_1 - b_1 A_2}{a_1 b_2 - a_2 b_1} \quad (41)$$

$$e_2 = \frac{a_1 A_2 - a_2 A_1}{a_1 b_2 - a_2 b_1} \quad (42)$$

$$A_1 = d_{AD}^k + \bar{v}_{AD}^k - d_{CD}^k - \bar{v}_{CD}^k + B_1 \quad (43)$$

$$A_2 = d_{BD}^k + \bar{v}_{BD}^k - d_{CD}^k - \bar{v}_{CD}^k + B_2 \quad (44)$$

$$A_3 = d_{CD}^k + \bar{v}_{CD}^k - B_3 \quad (45)$$

$$B_1 = \sum_{i=x,y,z} [(i_C^{k+1})^2 - (i_A^{k+1})^2] \quad (46)$$

$$B_2 = \sum_{i=x,y,z} [(i_C^{k+1})^2 - (i_B^{k+1})^2] \quad (47)$$

$$B_3 = -\sum_{i=x,y,z} (i_C^{k+1})^2 \quad (48)$$

Expression (20) can serve as a criterion for selecting the

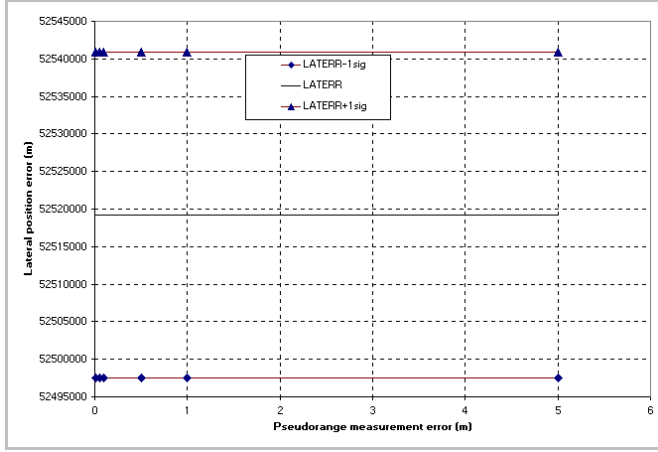


Fig. 5. Lateral position error vs. pseudorange measurement noise CLSA.

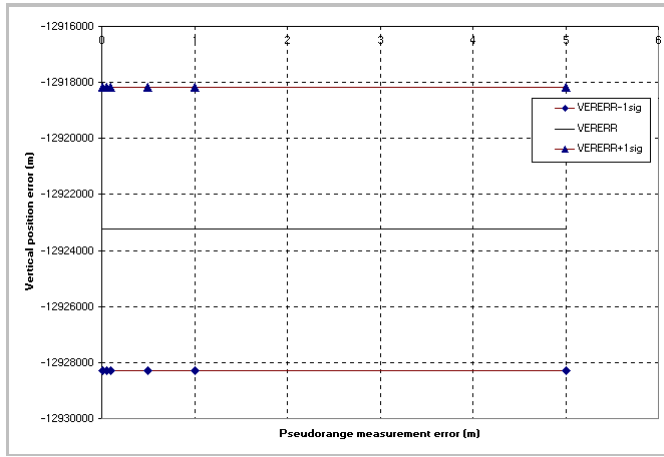


Fig. 6. Vertical position error vs. pseudorange measurement noise CLSA.

correct z_D^{k+1} .

2.4 Multidimensional noisy Applications

The expressions for the LSA solution utilizing DD pseudorange obtained for the multiple range sources is similar to that obtained in [9]. We take the time here, however, to explain some important differences of modifying the Doppler based navigation algorithm for satellites.

First, we note that the satellites' range vector from the center of the earth can be expressed based on the Taylor series expansion as,

$$\mathbf{R}[k+1] = \mathbf{R}[k] + \dot{\mathbf{R}}[k] + 0.5\ddot{\mathbf{R}}[k] + O(\ddot{\mathbf{R}}[k]) \quad (49)$$

where, $\mathbf{R}[k]$, $\dot{\mathbf{R}}[k]$, $\ddot{\mathbf{R}}[k]$, and $\ddot{\mathbf{R}}[k]$ are the satellite's range, range rate, the rate of range rate, and third derivative of range respectively, from the center of the earth at the k^{th} epoch. The quantity $O(\ddot{\mathbf{R}}[k])$ denotes the remainder of the Taylor series expansion for range derivative terms of orders higher than the third. Expression (49) assumes a 1-Hz data rate. A similar expression can be obtained for the range vector between the moving satellite and the user as,

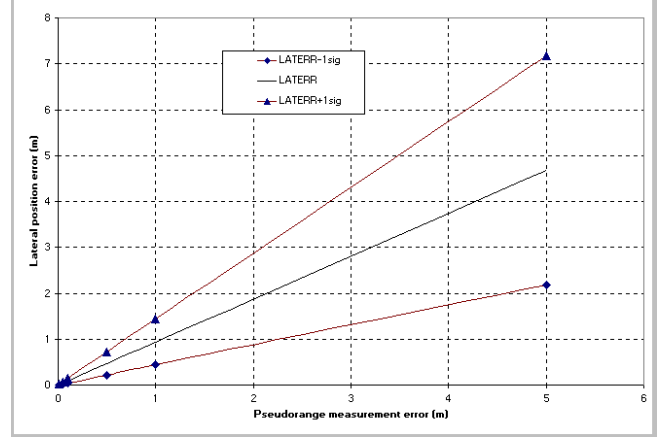


Fig. 7. Lateral position error vs. pseudorange measurement error MLSA.

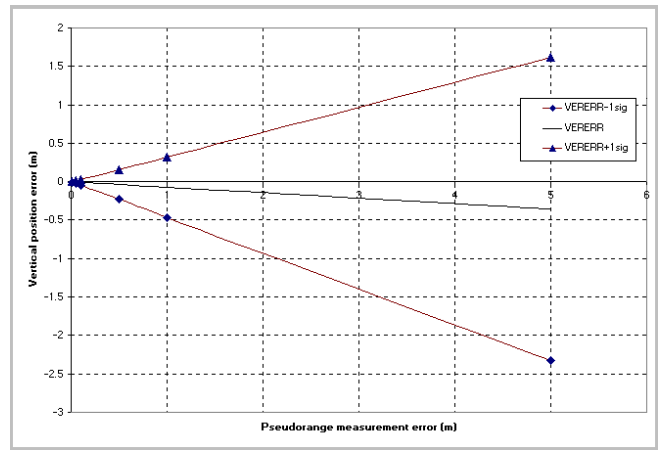


Fig. 8. Vertical position error vs. pseudorange measurement error MLSA.

$$\mathbf{R}_U[k+1] = \mathbf{R}_U[k] + \dot{\mathbf{R}}_U[k] + 0.5\ddot{\mathbf{R}}_U[k] + O(\ddot{\mathbf{R}}_U[k]) \quad (50)$$

where $\mathbf{R}_U[k]$, $\dot{\mathbf{R}}_U[k]$, $\ddot{\mathbf{R}}_U[k]$, and $\ddot{\mathbf{R}}_U[k]$ are the satellite's range, range rate, the rate of range rate, and third derivative of range respectively, from the user U at the k^{th} epoch. The only observable quantities of expression (50) are the range and the range rate utilizing any commercial receiver in the market. The challenge in this case is to derive a quantity from Doppler measurement imitating the raw pseudorange measurements. In order to accomplish that, we first estimate the range knowing the initial location of the receiver and the location of the satellites. Next, the range rate can be replaced with the Doppler measurements. The rate of the range rate can be approximated with successive differences of Doppler. Finally, a process noise sequence can serve as the remainder of the Taylor series expression $O(\ddot{\mathbf{R}}_U[k])$ (see (50)). The result of this work would be the expression for the Doppler derived pseudorange vector, which looks like,

$$\boldsymbol{\rho}_U[k+1] = \begin{bmatrix} \hat{\mathbf{R}}_U[k] + \boldsymbol{\phi}_U[k] \\ +0.5(\dot{\boldsymbol{\phi}}_U[k] - \dot{\boldsymbol{\phi}}_U[k-1]) + \mathbf{w}_U[k] \end{bmatrix} \quad (51)$$

Based on formulation (51) for the Doppler derived pseudorange

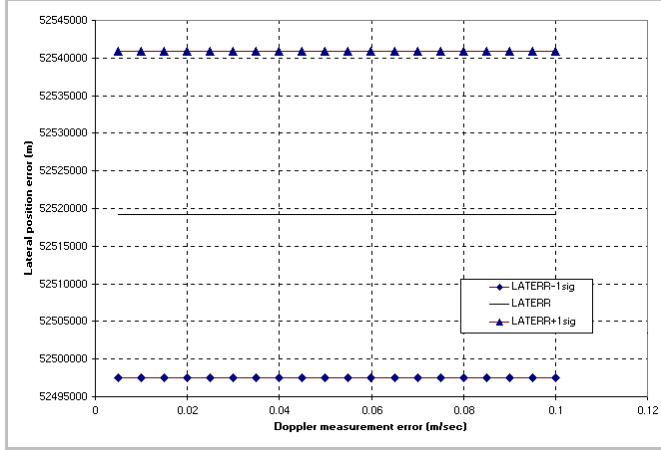


Fig. 9. Lateral position error vs. Doppler measurement error CLSA.

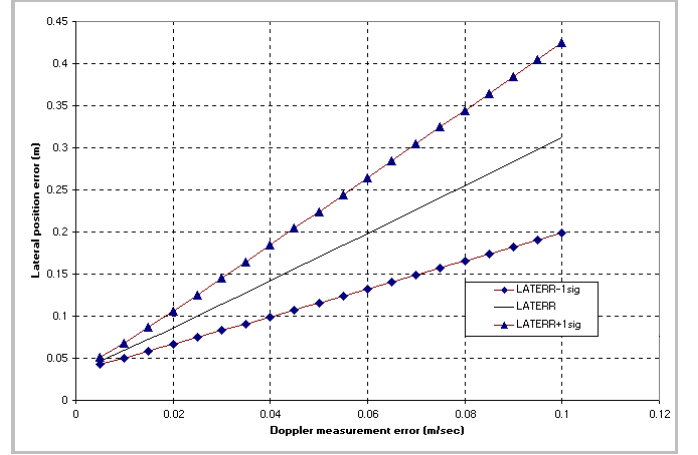


Fig. 11. Lateral position error vs. Doppler measurement noise MLSA.

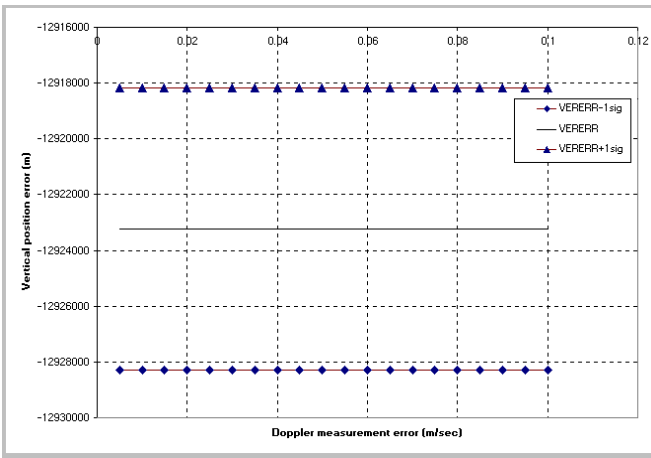


Fig. 10. Vertical position error vs. Doppler measurement noise CLSA.

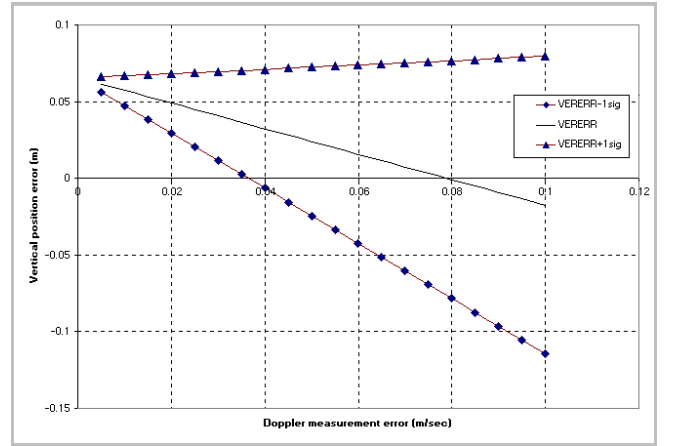


Fig. 12. Vertical position error vs. Doppler measurement noise MLSA.

we can derive an expression for the noise quantities as,

$$\sigma_p = 3\sigma_{\dot{\varphi}} + \sigma_w \quad (52)$$

The expression of Doppler derived pseudorange (51) is then used to compute the user solution for either the conventional or modified LSA [9].

2.5 The Impact of Error Sources

The impact of error sources affecting the Doppler measurement can be classified in four categories: ionosphere, troposphere, receiver measurement noise, and multipath. Although extensive study of the ionosphere and troposphere models are done by J.A. Klobuchar [14] and J.J. Spilker Jr. [15], a model which performs correction of both the ionosphere and troposphere effects for a single frequency receiver is yet to come.

The Doppler receiver measurement noise was estimated and the result of this work is shown in Tab. I, where $\bar{d}$ denotes the sample mean and S and standard deviation.

It is conceivable that the effect of the multipath and ionosphere, troposphere, and multipath are included in the Doppler measurement noise estimate.

TABLE I: ESTIMATED DOPPLER MEASUREMENT NOISE

Item	SAT1	SAT2	SAT3	SAT4
$\bar{d}$ (cm/s)	0	0	0	0
S (cm/s)	5	7	9	11

3 Simulation

Simulation results are provided for one moving GNSS receiver scenario.

3.1 Moving GNSS Receiver

The satellite constellation can be generated using either almanac data or ephemeris data [13]. However, for simplicity the simulation results were obtained for a GPS constellation driven by almanac data. The moving scenario, depicted in Fig. 4, was selected from one the indoor geo-location applications [9].

1. Raw pseudorange CLSA

First, we process the raw pseudorange using the CLSA and

obtain the lateral and vertical position error for pseudorange measurement error (1 sigma) ranging from 0.01 to 5 m (see Fig. 5 and 6).

2. Raw pseudorange MLSA

Next, we process the raw pseudorange measurements utilizing the modified least square algorithm [9], [16] (see Figs. 7 and 8).

3. DD pseudorange CLSA

Next, we process the Doppler derived pseudorange and obtain the lateral and vertical position error for Doppler measurement error (1 sigma) ranging from 0.01 to 0.1 m (see Figs. 9 and 10).

4. DD pseudorange MLSA

Here, we repeat case III only utilizing the MLSA algorithm and the results are presented in Figs. 11 and 12.

4 Conclusions

According to the analytical and simulation results of this work it appears that both positioning and navigation with Doppler measurements are possible even for indoor or outdoor pseudolite geolocation applications.

It appears that the CLSA algorithm provides erroneous lateral and vertical position for the user despite the pseudorange or Doppler measurement error. For a constellation of 10 satellites and a system PDOP of closer to 1, obviously the geometry is not the primary deteriorating source of the navigation solution. A detailed discussion of this phenomenon will be the object of another article. We just mention here that it appears from the simulation point of view that the method under observation is incorrect.

On the other hand, the MLSA algorithm appears to provide consistent results with pseudolite data only [9].

Thus, processing Doppler derived pseudorange and utilizing the MLSA algorithm yields a navigation solution *fifty times better; i.e., submeter lateral/vertical position and submeter per second lateral/vertical velocity errors*, than the pseudorange MLSA.

5 Acknowledgement

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I want to profoundly thank my Ph.D. advisor Prof. William

R. Michelson for his support for my Ph.D. dissertation on [17] which served as the foundation for my book on *Indoor Geolocation Systems—Theory and Applications. Vol. I* (Not yet available in print) [18].

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ⁱ *Editor's Note:* This article was originally published in 2001 [1]. The main results of this article are valid if only the data are assumed to be normally or Gaussian distributed. For other types of data distributions, I believe that a modification of the algorithm may be needed. The original publication contained a few typos and some minor language problems that I believe are fixed in this edition. A number of other clarifications are made in this

publication and also the equation editor is greatly improved. Since, the original article did not contain a DOI and it is a copyright by the authors I believe that the publication of this article in the form of a technical report is a good way to preserve the originality and authenticity of the article and provide a much better understanding of the results of this work.