

## *Technical Report*

# **Indoor Adaptive GNSS Signal Acquisition. Part 1: Theory and Simulations**

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Indoors, GNSS signal encounters severe multipath power loss and fading which leads to significant signal degradation of the amplitude and phase to perform GPS (or GNSS) signal acquisition. To overcome these effects, piling up received GPS (or GNSS) data is a traditional (or conventional) method; however, it exhibits two shortcomings (drawbacks or limitations): instable detection performance, such as the probability of false alarm (PFA) fluctuates due to changes of the signal-to-noise ratio (SNR); and elongated acquisition time caused by extended accumulation duration and over repetitive FA occurrence (or “penalty”). To overcome the first shortcoming, an adaptive structure is employed in GNSS signal acquisition that enables constant FA rate (CFAR) criteria to guarantee a stable detection performance on the GNSS signal acquisition. To overcome the second shortcoming, an adaptive determination on accumulation length is employed to minimize the accumulation duration; therefore, a double-dwell structure (DDS) is used to reduce the processing time “penalty” caused by FA. Simulation results illustrate that an adaptive, stable detection implementation and DDS reduce the average acquisition time by almost fifty percent (or by half or a factor of two).

*Index Terms*—Adaptive GNSS signal acquisition; weak GNSS signal; constant FA rate (CFAR); DDS; generalized modified Bessel function distribution; modified Bessel function; parabolic cylinder function.

## **1 Introduction**

In challenging environments, indoor, urban street, dense woods and foliage, etc., the GPS signal power level degrades severely

due to fading caused by shadowing, reflection, refraction, multipath effects, etc. (Proгри 2016, [1]; Proгри et al. 2007, [2]; Lachapelle et al. 2003, [3]; Proгри 2003, [4]). Therefore, received indoor GPS (or GNSS) signals are significantly

weaker in power (or distorted in amplitude and phase) than the received GPS signals in open space; hence, in the literature they are known as weak GPS (or GNSS) signals.

At the present time, the everyday existence of weak GPS (or GNSS) signals disrupts (or causes to malfunction) the everyday functionality (practicality or practice or performance or operation) of GPS (or GNSS) receivers because the later use traditional methods (or algorithms or systems) to both acquire and track GPS (or GNSS) signals under the assumption that the GPS (or GNSS) signals can be received in open space or exhibit minor power losses or signal degradations.

Most people in the literature have recognized that we have a major problem: GPS receivers do neither acquire nor track weak GPS signals. How can we solve this challenging or significant problem? Most elaborated detailed discussions on addressing this problem are provided in Proгри pioneer publication 2016 [1]. For the sake of space and argument we are going to limit the discussion on just improvements that are obtained from acquiring weak GPS (or GNSS) signals; i.e., GPS receiver acquisition and tracking algorithms (or methods).

There is good literature on various weak GPS signal acquisition methods that can roughly be divided into four main categories: the GPS data accumulation (Psiaki 2001 [5]; Ziedan and Garrison 2004 [6]; Elderts-Boll and Dettmar 2004 [7]; Yu et al. 2007 [8]); the GPS signal acquisition performance by mitigating the inference from noise, jamming signals or the cross-correlation results from unexpected GPS signals (Proгри 2011 [9]; Balaei and Dempster 2009 [10]; Brenneman et al. 2010 [11]; Deerga and Swamy 2006 [12]; Fante and Vaccaro 2000 [13]; Amin et al. 2004 [14]); the novel signal detection schemes (Huang et al. 2013 [15]; Huang and Pi 2009 [16]; Mahani et al. 2003 [17]); multi-user detection acquisition and tracking (Proгри et al. 2005 [18]; Proгри et al. 2009 [19]).

Psiaki 2001, [5] studied two weak GPS signal acquisition methods: “full-bit” and “half-bit” based on non-coherent accumulation (NCAC) that enables SNR improvement by piling up the GPS data or intuitively, by accumulating more GPS data in the condition that the carrier-to-noise ratio (CNR) is positive. However, there are some factors that limit the GPS data length extension: the square loss and the signal distortion caused by the Doppler shift. The longer the accumulated GPS data is generated the more severe the square loss which profoundly degrades the accumulation efficiency. Similarly, the longer the accumulated data length the more fluctuations caused by waveform distortions. Therefore the GPS signal

SNR improvement based on NCAC is limited to a certain range (or extent). Different from NCAC, the differential coherent accumulation (DCAC) method is also employed to acquire weak GPS signal (Elderts-Boll and Dettmar 2004 [7]; Yu et al. 2007 [8]; Wang et al. 2013 [20]; Borio et al. [21]). The differential accumulation method sums up the products of any two adjacent coherent accumulation (CAC) results. The differential accumulation method increases the tolerance on the Doppler shift and the data bit transition.

However, the SNR improved efficiency is not high enough compared to the CAC method in the case of the small Doppler shift.

Proгри, 2011 [9] proposed significant improvement of the signal to interference by restoring the phase of the desired signal; Balaei and Dempster, 2009 [10] studied the different statistics between the expected GPS signal and the interference and proposed an effective method of suppressing the jamming signal according to different statistics variables.

Brenneman et al, 2001 [11] employed smart antenna arrays to perform spatial filtering which suppresses signals on the secondary paths and improves the SNR. Deerga and Swamy, 2006 [12] suppressed the frequency modulation interference by the time-frequency technology. Fante and Vaccaro, 2000 [13] employed the antenna array to implement the space-time signal processing technology to cancel the wideband inference. Amin et al. 2004 [14] studied the orthogonality between the signal subspace and the noise subspace to mitigate the degradation by the noise and interference. Similarly, antenna arrays are employed in the subspace of GPS signal processing Proгри et al. 2009 [19].

Huang and Pi, 2009 [16] employed the Duffing chaotic oscillator to detect the weak GPS signal which utilizes the dynamic features of the chaotic oscillator, the sensitivity to the periodical signal, and the immunity to noise. Mahani, 2003 [17] used a cascading structure to handle the interference from the ground GPS signal amplifier.

One simplified and acceptable description of GPS signal fades is based on severe effects caused by two main factors: shadowing and multipath effects. Consider an indoor environment for example, the GPS signal strength decreases extremely fast after going through all kinds of medium, like window glass, concrete wall, wooden wall, etc. Furthermore, due to inconsistencies that lie in the medium material and path lengths of the transmission medium, the GPS signal strength (or power or amplitude or phase) fluctuates severely.

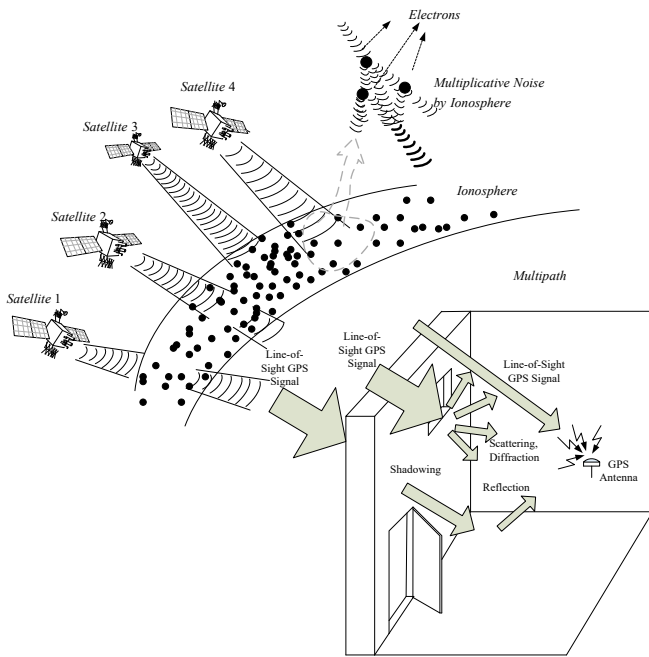


FIGURE 1: GPS signal fading model.

Therefore, the total distorted received GPS signal characteristics due to multipath continuously change due to independent changes on signal characteristics on each path respectively. Figure 1 presents a typical scenario of an indoor GPS signal degradation characteristics due to multipath and shadowing effects.

The two main fading factors, (shadowing and multipath) as illustrated in Fig. 1, cause the GPS signal receiver to stay either in the static status (or coast) or movement at extremely low speed due to the spatial frame. Actually, even when the GPS receiver stays in the static status, the movement of the GPS satellites also changes the signal angle of arrival which induces random carrier phase fluctuations. Therefore, the inconsistency of the transmission medium material, the independent changes on each path and the ever existing Doppler shift induce severe fluctuations on the GPS signal strength at the GPS antenna.

Although, the weak GPS signal acquisition methods, (Psiaki 2001 [5]; Ziedan and Garrison 2004 [6]; Elderts-Boll and Dettmar 2004 [7]; Yu et al. 2007 [8]) based on the GPS data accumulation, are studied for quite a long time, and although acquisition methods are mature and easy to be implemented; however, to the best of our knowledge, increased acquisition time expenditure caused by weak GPS signal acquisition has not been studied well. Therefore, the long first-acquisition-time (FAT) brings inconvenience to GPS users; thus, becoming an important index to evaluate GPS receivers.

Since, in conditions with light-of-sight (LOS) GPS signal a

GPS receiver can successfully acquire a GPS signal much faster than in the absence of a LOS GPS signal; hence, the fixed GPS signal acquisition scheme is not a good practice in designing GPS receivers that operate in applications extending to indoor, underground, or underwater Progri (or Progri et al.) [1], [2], [4], [9], [18]-[19], [22]-[24].

One way to overcome the absence of a LOS GPS signal during the GPS signal acquisition is by means of adaptive communication and radar receivers (Weiss 1982 [25]; Griep et al. 1994 [26]). Weiss, 1982 studied three types of CFAR adaptive signal detection schemes with different threshold setting principles [25]. Griep et al. 1994, [26] presented a direct sequence spread spectrum (DSSS) ranging system with the CFAR threshold setting scheme. However, the study of Weiss 1982 [25] did not pay enough attention to the special characteristics of spread spectrum signal which can hardly be employed to assist during GPS signal processing. Griep et al. 1994 [26] focused on scenarios that contain DSSS signals at high power levels, but neglected scenarios in which the DSSS signal stays in the weak or low power level.

Since FA continues to exist in adaptive GPS signal acquisition scheme, the conventional GPS receiver consumes a long time to re-enter the acquisition state if FA occurs continuously. The time expenditure on restarting the GPS signal acquisition significantly increases the average acquisition time. To reduce the average acquisition time caused by FA, DDS is used (Oh et al. 2000 [27]; Kim et al. 1998 [28]; Shin and Lee 2003 [29]). Oh et al (2000) and Kim et al 1998 [28] combined two non-coherent detection systems to detect DSSS signals. Shin and Lee 2003 [29] employed the non-coherent structure to assist the differential coherent structure just by providing the initial pseudorandom code carrier phase information. The existing DDS cannot be applied to the GPS signal acquisition directly without the discussion on GPS signal step-by-step search in the domain of the initial pseudorandom code phase included in this paper. In one classic example of the existing DDS which includes CAC taken as the first dwell followed by NCAC, searching of the Doppler shift was neglected; hence, the DDS has failed to utilize the advantages of different statistics. In this paper, the improved DDS GPS signal acquisition structure is proposed which combines NCAC and DCAC modules. The novel structure utilizes a large Doppler shift tolerance of DCAC which is different from the work by Oh et al. 2000 [27] and Kim et al. 1998 [28]. The combined 2-tier (NCAC and DCAC)

structure, do not work in the master-server relationship which is adopted in the work by Shin and Lee 2003 [29]. Before the DDS based GPS signal acquisition structure is proposed, the GPS signal detection threshold adaptive setting and the accumulation data length determination are discussed in this paper.

First, the NCAC based adaptive GPS signal detection structure is studied obeying CFAR. Second, the DCAC based adaptive detection scheme is analyzed, which also observes CFAR. Since the background noise power level estimation is very important in the adaptive GPS signal detection, the improvement on the background noise estimation method is presented. Third, to reduce the expenditure on acquisition time, the adaptive determination on the accumulated GPS data length is also studied; the DDS based GPS signal acquisition structure is proposed. Fourth, simulations and experiments are performed to test the effectiveness of the adaptive acquisition scheme and the advantages on the acquisition performance. The experimental results verify the effectiveness of the novel GPS signal acquisition scheme based on DDS; i.e., DDS reduce the average acquisition time by almost fifty percent (or by half or a factor of two).

This paper is organized as follows: in Sect. 2 CFAR on adaptive GPS signal acquisition based on NCAC is discussed. Adaptive GPS signal detection based on DCAC is discussed in Sect. 3. Background noise power estimation in indoor adaptive GPS signal detection is presented in Sect. 4. Adaptive change on data length of NCAC is depicted in Sect. 5. Adaptive GPS signal detection based on DDS is shown in Sect. 6. Section 7 contains numerical results; Conclusion is provided in Sect. 8 along with a list of references. In Appendix A, probability of misdetection is discussed. In Appendix B, DDS Markov chain analyses are presented.

## 2 CFAR on Adaptive GPS Signal Acquisition Based on NCAC

In this section, the statistics of CAC on GPS signals L1 C/A code, L1C, and Galileo E1 OS, the principle of CFAR adaptive GPS signal detection based on CAC and CFAR GPS signal detection based on NCAC are discussed.

### 2.1 Statistics of CAC on GPS Signals L1 C/A code, L1C, and Galileo E1 OS

Consider  $K$  visible GPS satellites in sight. The GPS C/A-

code waveform transmitted from a  $k$ th GPS satellite  $x_k(t_k)$  is given by Progi [1].

$$x_k(t_k) = c_k(t_k)d_k([t_k]_D)e^{j(2\pi L_i t_k + \theta_{k0})} \quad (1)$$

where  $t_k$  is the time of transmission of the  $k$ th GPS signal;  $c_k(t_k)$  is pseudorandom code at time  $t_k$  which for the GPS L1, L2, or L5 [2] is the C/A code with code repetition sequence period at 1 ms,  $L_i$  is carrier frequency which for GPS is the  $L1 = 1575.42$  MHz,  $L2 = 1227.6$  MHz, or  $L5 = 1176.42$  MHz,  $d_k([t_k]_D)$  is the data bit transition for satellite  $k$  spread by the code  $c_k(t_k)$  [1] and  $D$  is the period when a data bit transition occurs (for the GPS L1 data case, it is equal to 20 ms),  $\theta_{k0}$  is some initial carrier phase of the signal; and the index  $k$  changes from  $\{1, 2, \dots, K\}$ .

While the total number of GPS satellites is close to thirty one as of 2010 [1] the total number of visible GPS satellites is typically twelve or around ten depending on your location and the visibility of your location; i.e., the GPS antenna.

The GPS L1C baseband signal contains seventy five percent of the power in the pilot signal and twenty five percent of the power in the data signal which can be written as [1].

$$s_{L1C_D}(t) = PC_{L1C_D}(t)D(t)BOC_{(1,1)}(t) \quad (2)$$

$$s_{L1C_P}(t) = \sqrt{3}PC_{L1C_P}(t)SC_{L1C_P}(t)TMBOC_{(6,1,4/33)}(t) \quad (3)$$

with the Primary Code (PC), Secondary Code (SC), navigation data (D), and TMBOC defined as

$$TMBOC_{(6,1,4/33)}(t) = \alpha(t)BOC_{(1,1)}(t) + \beta(t)BOC_{(6,1)}(t) \quad (4)$$

where

$$\alpha(t) = \begin{cases} 1, & t \in 29/33 \\ 0, & t \in 4/33 \end{cases} = \begin{cases} 1, & 0,0,0,1,0,1,0,0,0,0,0,0,0,0,0,0 \\ 0, & 0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0 \end{cases} \quad (5)$$

$$\beta(t) = 1 - \alpha(t) = \begin{cases} 0, & t \in 29/33 \\ 1, & t \in 4/33 \end{cases} \quad (6)$$

In the case of the Galileo E1 OS, the MBOC is used instead of TMBOC, and other signal design details can be obtained in Progi 2016 [1].

Progi *et al* [18] explains brilliantly all the steps required to obtain the received GPS signal via (1) through (16) in [18]. The only addition to Progi's model is the inclusion of the multiplicative noise [18]. Therefore, the received GPS signal degraded by additive noise and multiplicative noise simultaneously can be expressed as follow at the discrete index,  $n$ , as

$$r(n) = [x(n) \equiv \sum_{k=1}^K c_k(\tilde{\tau}_k) d_k([n]_D) e^{j\varphi_k} \varepsilon_+(n)] + \varepsilon_+(n) \quad (7)$$

where

$$\tilde{\tau}_k = nT_s - \tau_k, \text{ and } \varphi_k = 2\pi f_k \tilde{\tau}_k \quad (8)$$

where,  $f_k$ , denotes the  $k$ th visible GPS satellite Doppler frequency,  $T_s$  is the sampling period in seconds (s),  $\tau_k$  is the residual code phase delay,  $\varepsilon_+(n)$  denotes multiplicative noise and  $\varepsilon_+(n)$  denotes additive Gaussian noise. The multiplicative noise is generated by the abundant electrons in the ionosphere and high density of multipath. And the additive noise is usually considered to be induced by the receiver hardware [15].

In order to perform acquisition of a GPS signal, correlation between the received GPS signal,  $x(n)$ , and locally generated identical replica,  $l(n)$ , corresponding to the  $k$ th GPS satellite signal, is performed; where, assuming that there is no Doppler effect on the locally generated signal we have

$$l(n) = c_k(nT_s) \quad (9)$$

For simplification in the later analysis, the chip length distortion effect and the data bit are ignored. Simplifying on GPS signal analytical expression is reasonable because the chip length distortion can be compensated by the local GPS signal replica distorted by Doppler shift. And the data bit caused carrier phase transition can be avoided by algorithms. Furthermore, the research in this paper focuses on the case in which multiplicative noise is small enough and can be ignored. The processing method which handling GPS signal degraded by multiplicative and additive noise is analyzed Huang et al, 2013 [15]. Based on these above assumptions, (1) can be rewritten as

$$r(n) = \sum_{k=1}^K A_k c_k(\tilde{\tau}_k) e^{j(\varphi_k + \theta_{k0})} + \varepsilon_+(n) \quad (10)$$

Due to the orthogonality of the pseudo random code, the received GPS signal from only one satellite is taken into consideration in our analysis. In the correlation-based acquisition scheme, the adjacent (or remaining) GPS signals can be considered as additive noise to the desired (or received) GPS signal.

Derived from the assumption above, the received GPS signal from one satellite can be rewritten as:

$$r_k(n) = A_k c_k(\tilde{\tau}_k) e^{j(\varphi_k + \theta_{k0})} + \varepsilon_+(n) \quad (11)$$

GPS signal in 1 ms can be expressed below in the condition of 5 MHz sampling frequency [30]

$$\mathbf{r} = A_k \begin{bmatrix} c_k(0T_s - \tau_k) e^{j[2\pi f_k(0T_s - \tau_k) + \theta_{k0}]} + \varepsilon_+(0) \\ c_k(1T_s - \tau_k) e^{j[2\pi f_k(1T_s - \tau_k) + \theta_{k0}]} + \varepsilon_+(1) \\ \vdots \\ c_k[L_-T_s - \tau_k] e^{j[2\pi f_k[L_-T_s - \tau_k] + \theta_{k0}]} + \varepsilon_+(L_-) \end{bmatrix} \quad (12)$$

where

$$L_- = L - 1 \quad (13)$$

where  $T_s$  is sampling period,  $T_s = 1/5000$  ms and  $L = 5000$  [30] which is also good for the GPS L1C code and Galileo E1 OS. Locally generated C/A code in one period can be expressed as:

$$\mathbf{s} = \begin{bmatrix} c_k(0T_s) e^{j2\pi f_k(0T_s)} \\ c_k(1T_s) e^{j2\pi f_k(1T_s)} \\ \vdots \\ c_k[L_-T_s] e^{j2\pi f_k[L_-T_s]} \end{bmatrix} \quad (14)$$

The maximum value  $x$  of the circular correlation results between  $\mathbf{r}$  and  $\mathbf{s}$ ,

$$x = \mathbf{r}' \mathbf{s}^* = \left| \sum_{l=0}^{L-1} \left[ c_k(lT_s - t_0) c_k(lT_s) e^{j(2\pi l T_s f_d + \theta)} + c_k(lT_s) e^{j(2\pi l T_s f_0 + \theta)} \varepsilon_+(lT_s) \right] \right| = \left| \Delta(t_0) \frac{\sin(\pi L T_s f_d)}{\sin(\pi T_s f_d)} \frac{e^{j\pi L T_s f_d}}{e^{j\pi T_s f_d}} + \sum_{l=0}^{L-1} c_k(lT_s) e^{j(2\pi l T_s f_0 + \theta)} \varepsilon_+(lT_s) \right| \quad (15)$$

Where  $\Delta(t_0)$  is the pseudorandom code correlation function versus time, and  $0 < \Delta(t_0) < \frac{1}{2} \frac{1}{5000}$  ms.

Since the additive Gaussian noise  $\varepsilon_+(t)$  is independent at different time point  $t$ , under the assumption  $H_1$  with the existence of GPS signal,  $x$  observes Gaussian distribution. The

mean is  $E(x) = \Delta(t_0) \frac{\sin(\pi L T_s f_d)}{\sin(\pi T_s f_d)} \frac{e^{j\pi L T_s f_d}}{e^{j\pi T_s f_d}} = \mu_{x|H_1}$  and the variance is  $\sigma_x^2 = L\sigma_{\varepsilon_+}^2$ . Under the assumption without the existence of GPS signal,  $x$  can be assumed to be a Gaussian distribution with mean  $E(x) = \mu_{x|H_0} = 0$  and variance  $\sigma_x^2 = L\sigma_{\varepsilon_+}^2$ .

To normalize the variance of  $x$  the correlation peak value  $x$  is written as:

$$x' = \frac{1}{\sigma_x} x; \mu'_{x|H_1} = \frac{1}{\sigma_x} \mu_{x|H_1}; x'' = x' - \mu'_{x|H_1} \quad (16)$$

The probability density function (pdf) of  $x'$ , under assumption  $H_i$ ,  $i = \{0,1\}$ , is normal of Gaussian [1]

$$p(x^i|H_i) = \frac{1}{\sqrt{2\pi}} e^{-0.5x'^2}; i = \{0 \leftrightarrow ', 1 \leftrightarrow ''\} \quad (17)$$

To guarantee CFAR, the following should be satisfied,

$$P_{fa} = \int_{\varrho}^{\infty} p(x'|H_0) dx' \quad (18)$$

Where  $\varrho$  denotes the threshold; hence, (18) can be rewritten with the help of the complementary error function,  $\text{erfc}(x)$ , as [31]

$$P_{fa} = \int_{\varrho}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-0.5x'^2} dx' = \text{erfc}(\varrho) \quad (19)$$

The threshold,  $\varrho$ , can be obtained by means of the inverse complementary error function,  $\text{erfc}^{-1}(x)$ , as

$$\varrho = \text{erfc}^{-1}(P_{fa}) \quad (20)$$

## 2.2 Principle of CFAR Adaptive GPS Signal Detection Based on CAC

In order to detect a GPS signal, the absolute value  $x'_a$  of  $x'$  is used to compared with threshold to determine the existence of GPS signal,  $x'_a = |x'_{ai}|$ . Therefore, under the assumption,  $H_1$ ,  $x'_a$  follows a Rice distribution, the following is obtained

$$p(x'_a|H_1) = \frac{x'_a}{\sigma'^2_x} e^{-\frac{x'^2_a + \mu'^2_{x|H_1}}{2\sigma'^2_x}} I_0\left(\frac{x'_a \mu'_{x|H_1}}{\sigma'^2_x}\right) \quad (21)$$

Under assumption  $H_0$ ,  $x'_a$ , follows Rayleigh distribution, yields

$$p(x'_a|H_0) = \left(x'_a \equiv \frac{x''_a}{\sigma'^2_x}\right) e^{-0.5x''_a} \quad (22)$$

To guarantee CFAR, the following should be satisfied,

$$P_{fa} = \int_{\varrho}^{\infty} p(x'_a|H_0) dx'_a \quad (23)$$

Where  $\varrho$  denotes the threshold; hence, (23) can be rewritten as,

$$P_{fa} = \int_{\varrho}^{\infty} x''_a e^{-0.5x''_a} dx'_a \quad (24)$$

The threshold,  $\varrho$ , can be resolved from

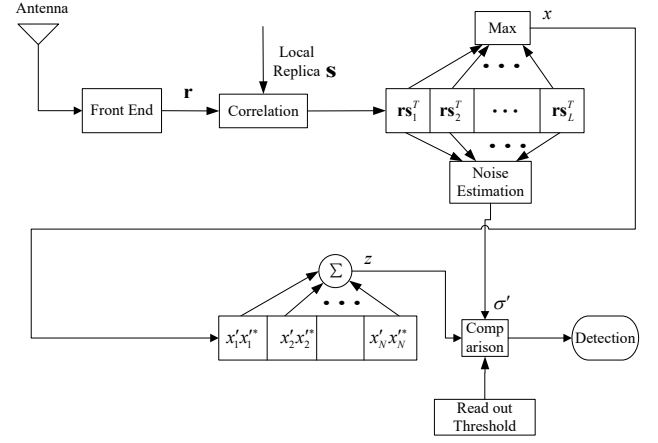
$$\varrho = -2\sigma'^2_x \log(P_{fa}) \quad (25)$$

## 2.3 Principle of CFAR Adaptive GPS Signal Detection Based on CAC

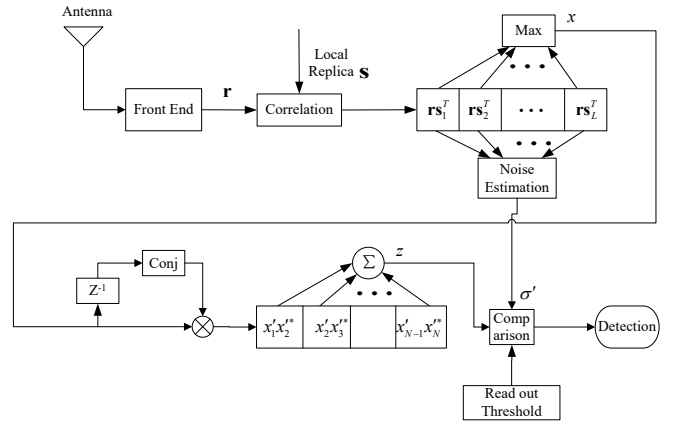
Figure 2 (a)/(b) depict the block diagram of an adaptive GPS signal detection based on NCAC/DCAC.

In order to detect the GPS signal, the SNR gain caused by CAC can be calculated by the formula below

$$G_{co} = 10\log(L_c) \quad (26)$$



(a) NCAC



(b) DCAC

FIGURE 2: Adaptive GPS signal detection block diagram based on (a) non-coherent (b) DCAC.

where  $L_c$  denotes the length of pseudorandom code, in ms. Indeed, limited by the carrier phase transition caused by data bit transition, the length of coherent accumulation data is no more than 10 ms. In challenging environments where GPS signal strength degrades severely, coherent integration cannot meet the initial GPS signal acquisition requirement. Fortunately, NCAC can be used to further increase SNR.

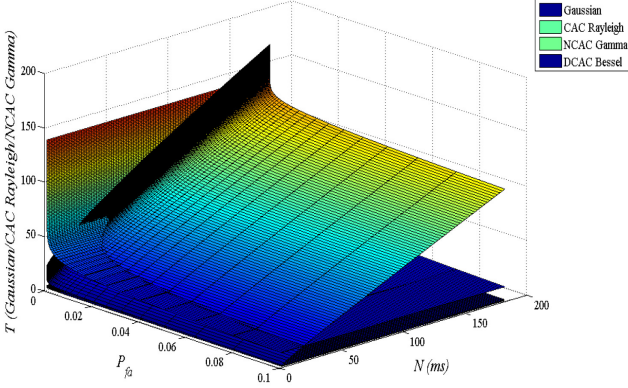
The GPS NCAC statistics is expressed below

$$z = \sum_{k=1}^N x_k x_k^* = \mathbf{x}^\dagger \mathbf{x}^* \quad (27)$$

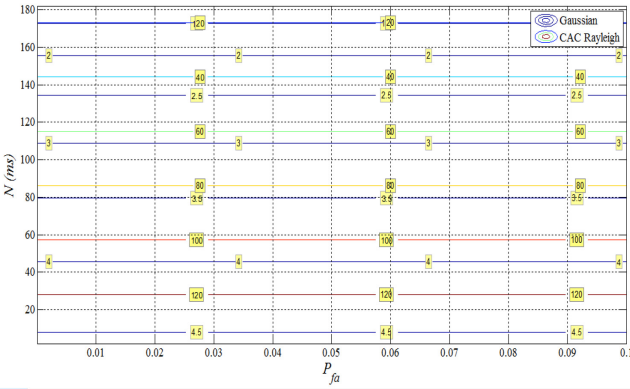
where  $\mathbf{x} = \frac{[x_1, x_2, \dots, x_N]^\dagger}{\sqrt{N}}$ ;  $x_k (k = 1, 2, \dots, N)$  denotes the CAC result for the length  $l$  ms and  $\dagger$  denotes the transpose operator. In common cases;  $l$  is a positive integral less than 10 ms. After the normalization processing on the variance of CAC result,  $x_k$ , the NCAC statistics  $z'$  can be expressed as

$$z' = \sum_{k=1}^N x'_k x'^*_k = \mathbf{x}'^\dagger \mathbf{x}'^* \quad (28)$$

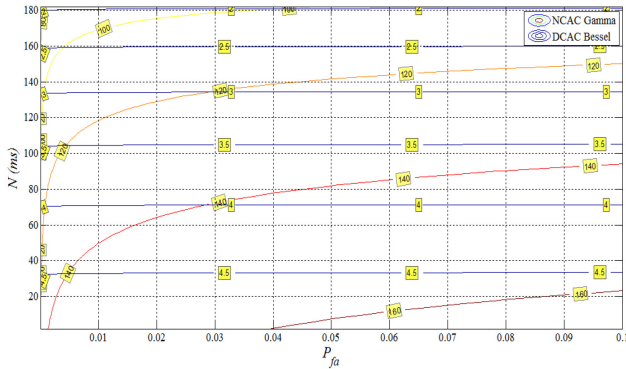
where



(a) 3-D Gaussian/CAC Rayleigh/NCAC Gamma/DCAC Bessel<sup>1</sup>



(b) 2-D Gaussian/CAC Rayleigh



(c) 2-D NCAC Gamma/DCAC Bessel

FIGURE 3: Threshold plane versus CNR and PFA: NCAC (a) 3-D all of them; (b) 2-D Gaussian/CAC Rayleigh; and (c) 2-D NCAC Gamma/DCAC Bessel.

$$\mathbf{x} = \frac{[x_1, x_2, \dots, x_N]^T}{\sqrt{NL \cdot \sigma_n}} \quad (29)$$

Under the assumption,  $H_0$ , statistics,  $z'$ , follows a Gamma pdf [1], [32]

$$p(z'|H_0) = \frac{1}{2^M [\Gamma(M) \equiv (M-1)! : N \equiv 2M]} z'^{M-1} e^{-0.5z'} \quad (30)$$

To achieve CFAR adaptive detection scheme, (28) should stand

$$P_{fa} \equiv \int_0^\infty p(z'|H_0) dz' = C = Q(M, Th) = \frac{\Gamma(M, Th)}{\Gamma(M)} \ll 1 \text{ iii} \quad (31)$$

where  $Q(a, b)$  is the regularized incomplete upper Gamma function [1] and  $\Gamma(M)$  is the gamma function [33]. The value of  $Th$  determines the threshold value of a GPS receiver, whose analytic value in (31) can be computed in closed-form using the definition of the inverse incomplete Gamma function as [34]

$$\varrho = Q^{-1}(P_{fa}, M) \quad (32)$$

Figure 3 (a)/(b)/(c) demonstrates the threshold plane versus PFA and length using the MATLAB definition of the inverse incomplete Gamma function [35] `gammaincinv` [34] for Gaussian/CAC Rayleigh/NCAC Gamma/DCAC Bessel. As shown in Fig. 3 (a)/(b), the mapping relationship from  $P_{fa}$  and NCAC length to threshold values is obvious. When designing a GPS receiver, first we estimate background noise power level, second, the NCAC length is determined; finally, the GPS detection threshold can be estimated using the inverse incomplete Gamma function (32), in contrast to computed analytical solution (25).

### 3 Adaptive GPS Signal Detection Based on DCAC

The outline of DCAC can be depicted as: compute the CAC result  $x_k$  and  $x_{k+1}$  from the received GPS signal corresponding to  $k$ th and  $(k+1)$ th respectively; perform conjugate multiplication between  $x_k$  and  $x_{k+1}$  and pile up the products to get the differential accumulation statistics

$$z_D = \sum_{k=1}^{N-1} x_k x_{k+1}^* \quad (33)$$

Under the assumption  $H_0$  (no GPS signal), statistics  $z_D$  follows a pdf of the Bessel function of second kind (see Progni 2016, [36] (58)),

$$p(z_D|H_0) = \frac{|z_D|^{0.5(P-2)} e^{-0.5|z_D|} K_{0.5(P-2)}(|z_D| \sqrt{\omega_{11}\omega_{22}})}{\sqrt{\pi} \sqrt{|M_2|} (m_{11}m_{22})^{0.5P} \Gamma[0.5(P-1)]} \quad (34)$$

where  $\Gamma(\cdot)$  is Gamma function;  $K_{0.5(P-2)}(\cdot)$  is  $0.5(P-2)$  order Bessel function of second kind; and  $M_2$  is a normalizing constant given by

<sup>1</sup> For this plot  $T \equiv \varrho$  or  $T \equiv \rho$ .

$$|M_2| = \frac{(P-2)^2 2^{6P+2} \Gamma^2 \left( P-1, \frac{P}{2}, \frac{1}{2}, \frac{P}{2}, \frac{1}{2}, \frac{\omega_{12} - \sqrt{\omega_{11}\omega_{22}}}{\omega_{12} + \sqrt{\omega_{11}\omega_{22}}} \right)}{\pi(P-1)^2 (\omega_{11}\omega_{22})^{\frac{P}{2}} \left( \frac{\omega_{12}}{\sqrt{\omega_{11}\omega_{22}}} + 1 \right)^{2P-2} (m_{11}m_{22})^{\frac{P}{2}}} \quad (35)$$

Since white Gaussian statistics  $x_k$  and  $x_{k+1}$  are not correlated to each other; hence,  $m_{11} = m_{22} = (P-1)\sigma_x^2 = \sigma_x'^2$ ,  $\omega_{11} = \omega_{22} = \frac{1}{\sigma_x'^2}$ ,  $\omega_{12} = \omega_{21} = 0$ ; since  $P-1$  is a positive integer, then the normalizing constant  $M_2$  is given by

$$|M_2| = \frac{2^{P-2} \Gamma^2 \left( \frac{P-1}{2} \right)}{(P-1)^P} \equiv \frac{2^{2(M-1)} \Gamma^2 \left( \frac{M-1}{2} \right)}{(2M-1)^{2M}} \equiv \frac{2^{2L} \Gamma^2(L)}{(2L+1)^{2(L+1)}} \quad (36)$$

Hence, since  $z_D$  is the inner product result,  $z_D$  is a positive number (34) can be simplified as the pdf of the half modified Bessel function of the second kind (see Proгри 2016, [36] (71)),

$$p(z_D|H_0) = \begin{cases} \frac{z_D^L K_L \left( \frac{z_D}{\sigma_x'^2} \right)}{\sqrt{\pi} 2^{L-1} \Gamma \left( L + \frac{1}{2} \right) (\sigma_x'^2)^{L+1}}, & z_D \geq 0 \\ 0, & z_D < 0 \end{cases} \quad (37)$$

and  $P = 2M = 2(L+1) = N-1$ .

Under the assumption  $H_1$  (with GPS signal), statistics,  $z_D$ , in (33), has a parabolic cylinder pdf (see Proгри 2016 [37]) as follows

$$p(z_D|H_1) = \frac{d_0 \sum_{k=0}^{N-1} H_k(N)(N-k-1)! [b_0 D_{k-N}(c_0) + b_1 D_{k-N}(c_1)]}{\sigma_{z_D|H_1}^{N+k} C} \quad (38)$$

where  $D_k(z)$  is called the parabolic cylinder function (see [38] pg. 1028 9.24-9.25) and  $H_k(N)$  is the Hermite polynomials (see [38] pg. 996 8.95)

$$a_0 = \frac{z_D - \mu_{z_D|H_1}}{2[\sigma_{z_D|H_1}^2 \equiv a^2]}; \quad a_{0-} = \frac{1}{\sigma_{x|H_1}^2 \equiv c^2} - a_0; \quad a_{0+} = \frac{1}{\sigma_{x|H_1}^2} + a_0 \quad (39)$$

$$b_0 = e^{\frac{\sigma_{z_D|H_1}^2}{4} a_{0-}^2}; \quad b_1 = e^{\frac{\sigma_{z_D|H_1}^2}{4} a_{0+}^2} \quad (40)$$

$$c_0 = \sigma_{z_D|H_1} a_{0-}; \quad c_1 = \sigma_{z_D|H_1} a_{0+} \quad (41)$$

$$d_0 = e^{-a_0(z_D - \mu_{z_D|H_1})} \quad (42)$$

$$\mu_{z_D|H_1} = \mu_{x_k|H_1} \mu_{x_{k+1}|H_1} \quad (43)$$

$$\sigma_{z_D|H_1}^2 = \sigma_{x|H_1}^2 \sum_{k=1}^N \mu_{x_k|H_1}^2; \quad \sigma_{x|H_1}^2 = \sigma_{x_k|H_1}^2 \quad (44)$$

$$D_k(z) = 2^{\frac{1+k}{2}} W_{\frac{1+k}{2}, \frac{1}{4}} \left( \frac{z^2}{2} \right) z^{-\frac{1}{2}} \quad (45)$$

$$\frac{1}{C} = \frac{2^N \sigma_w \sigma^2 N}{\sqrt{2\pi(N-1)!}} = \frac{e^{-\frac{a^2}{3c^4} \left( \frac{2a}{\sqrt{3}} \right)^{-N}} a^{-1}}{2\sqrt{2\pi} \sum_{k=0}^{N-1} H_k(N)(N-1-k)! \left( \frac{\sqrt{3}}{2a} \right)^k D_{k-N} \left( \frac{2a}{\sqrt{3c^2}} \right)} \quad (46)$$

and  $W_{k,l}(z)$  is Whittaker function [39] (see also [38] pg. 1024 9.22-9.23).

PFA,  $P_{fa}$ , can be computed from

$$P_{fa} = \int_0^\infty \frac{z_D^L K_L \left( \frac{z_D}{\sigma_x'^2} \right) dz_D}{\sqrt{\pi} 2^{L-1} \Gamma \left( L + \frac{1}{2} \right) (\sigma_x'^2)^{L+1}} \quad (47)$$

After normalizing the variance of statistics,  $z_D$ , (47) can be written as,

$$P_{fa} = \int_0^\infty \frac{|z'|^L K_L(z') dz'}{\sqrt{\pi} 2^{L-1} \Gamma \left( L + \frac{1}{2} \right)} \cong \text{erfc} \left( \frac{\rho}{2\sqrt{L}} \right) \text{iv} \quad (48)$$

where  $\rho = \varrho \sigma_x'^{-2}$ .

Since the analytic solution to (48) is very laborious to be computed analytically as shown in Proгри 2016 [36], it is shown however, that it approaches a half normal distribution with variance  $\sim 2L$  (see (75) in Proгри 2016 [36]) with an absolute error of the pdf and cdf within five percent, which can be employed to computed very easily the detection threshold,  $T$ . The threshold value plane versus  $N$  and PFA is presented in Fig. 3 (c). As shown in Fig. 3 (c) the DCAC has just as good performance in an indoor presence of additive and multiplicative noise as if it were in the ideal conditions of just signal and Gaussian noise.

## 4 Background Noise Power Estimation in Indoor Adaptive GPS Signal Detection

In this section the following are discussed: conventional background noise estimation method; and improvement on background noise power level for GPS signal.

### 4.1 Conventional Background Noise Estimation Method

From the discussions in the former sections, to implement adaptive GPS signal detection, the estimated background noise

<sup>2</sup> There are neither numerical issues with the constant  $C$  nor in general with the parabolic cylinder pdf of cdf (38); nevertheless, the parabolic cylinder cdf (38) needs to be computed in closed form and verified whether it is a valid cdf and its inverse needs to be computed also; hence, it is still under investigation

and it will require a lot of work to complete this investigation. The reader is encouraged to read Proгри 2016 [37] for an introduction of the parabolic cylinder pdf and  $C$ .

power is needed. One of the conventional and effective methods is the estimation based on the sliding window.

Figure 4 illustrates the outline of the sliding window-based noise level estimation method. In the mesh plot of Fig. 4, each cell denotes the correlation result between received GPS signal and local replica. The basic task in GPS signal acquisition lies in estimating the initial phase of pseudorandom code and Doppler shift; a.k.a. the 2-D GPS signal search. Since the maximum correlation value corresponds to the case when locally estimated initial pseudorandom code phase and Doppler shift most approach the actual values. Therefore, search the maximum value in the mesh plot first; after that, the average around the maximum value is calculated. To increase the estimation accuracy, a large number of cell samples for calculating background noise level should be guaranteed. This principle can be explicitly outlined as follows: use only eight sample points; repeat the average value computation around different central points to get different average values; calculate the variance of the different average values which equals to the noise power level. The principle expressed in (33) is revisited here as

$$x'_{ave}(k, u) = \frac{1}{M \cdot N} \sum_{i=k-M/2}^{k+M/2} \sum_{j=u-N/2}^{u+N/2} x(i, j) \quad (49)$$

where  $i$  and  $j$  do not equal to zero simultaneously. Repeat (49) and calculate the variance of  $x'_{ave}$  which is taken as the background noise power level. After the noise power level is fixed, the adaptive GPS signal detection scheme in the former section can be utilized.

#### 4.2 Conventional Background Noise Estimation Method

The traditional background noise power level estimation method, which calculates the average power level, is effective in processing signal at a single frequency. However, for noise estimation in GPS signal acquisition, improvement is needed. Narrow band signals at different single frequencies are orthogonal to each other. *For GPS signals at the same bandwidth and frequency, the pseudorandom code cannot be guaranteed to be orthogonal to each other, which is the reason why the background noise power level estimation method should be modified.*

The pseudorandom codes in GPS signal are not totally orthogonal to each other, which means the correlation result between different GPS signals does not equal to zero and autocorrelation value of the same pseudorandom code with

time shift does not equal to zero. In the range of 1 ms initial pseudorandom code phase shift, the autocorrelation equals to  $-65/1023$  of the maximum correlation with occurrence 12.5 %,  $63/1023$  of the maximum correlation with 12.5 % and  $-1/1023$  with seventy five percent. Using traditional background noise power level estimation method, the non-zero autocorrelation results of GPS signal are also taken as part of the background noise. Therefore, an improvement of the existed background noise is needed.

The improvement focuses on eliminating the “fake noise” caused by the mismatched pseudorandom code phase autocorrelation, which is called “fake noise” cancellation.

A 2-step procedure is needed to cancel “fake noises”: estimate background noise in the range of one chip pseudorandom code around the maximum correlation result; test the estimated background noise level using the data beyond one chip of pseudorandom code.

Let us suppose that  $A$  denotes the amplitude of GPS signal, and  $\sigma'^2$  is the variance of real background noise.

Figure 5 demonstrates the autocorrelation results curve versus initial pseudorandom code phase shift. The slopes are 1023 and  $-1023$  for the left and right part of the line in Fig. 5 respectively. Suppose the sampling frequency is 5 MHz, there are at least four points locating in the range of one chip of pseudorandom code. The left part and right part of the curve can be analytically described as

$$A(1023 \mp 1023 \times i) + \sigma'^2 = x_{M \pm i}; i = \{0, 1, 2, 3\} \quad (50)$$

The amplitude  $A$  and noise variance  $\sigma'^2$  can be estimated by calculating generalized inverse matrix of the coefficient matrix for over-determined (50), which are  $(\hat{A}_1, \hat{\sigma}'^2_1)$  and  $(\hat{A}_2, \hat{\sigma}'^2_2)$ .

Since the uncertain location of exact maximum autocorrelation result caused by sampling ambiguity, verification is needed to test amplitude,  $A$ , and noise variances,  $\sigma'^2$ . To verify the estimated amplitude and noise variance, the average autocorrelation point beyond the fifth sampling point from maximum autocorrelation is needed to be calculated, which is depicted in

$$\bar{x} = \frac{\sum_{k=1}^N x_k + \sum_{k=M-5}^{M+5} x_k}{N-11} \quad (51)$$

Furthermore, autocorrelation results do not equal to zero if the received GPS signal mismatch the local replica in initial code phase, as shown in (50), can be written as

$$\bar{x}(\hat{A}, \hat{\sigma}'^2) = \hat{A}^2 \left( \frac{65 \times 0.125}{1023} + \frac{63 \times 0.125}{1023} - \frac{1 \times 0.75}{1023} \right) + \sigma'^2 \quad (52)$$

| Initial Pseudorandom Code Phase |          |                                                     |          |          |          |
|---------------------------------|----------|-----------------------------------------------------|----------|----------|----------|
| $x(1,1)$                        | $x(1,2)$ | $x(1,3)$                                            | $x(1,4)$ | $\dots$  | $x(1,N)$ |
| $x(2,1)$                        | $x(2,2)$ | $x(2,3)$                                            | $x(2,4)$ | $\dots$  | $x(2,N)$ |
| $x(3,1)$                        | $x(3,2)$ | $\sum_{i=n-1}^{n+1} \sum_{j=n-1}^{n+1} x(i,j)$<br>8 | $x(3,4)$ | $\dots$  | $x(3,N)$ |
| $x(4,1)$                        | $x(4,2)$ | $x(4,3)$                                            | $x(4,4)$ | $\dots$  | $x(4,N)$ |
| $\vdots$                        | $\vdots$ | $\vdots$                                            | $\vdots$ | $\ddots$ | $\vdots$ |
| $x(M,1)$                        | $x(M,2)$ | $x(M,3)$                                            | $x(M,4)$ | $\dots$  | $x(M,N)$ |

Doppler Shift

FIGURE 4: Illustration of sliding window-based noise level estimation.

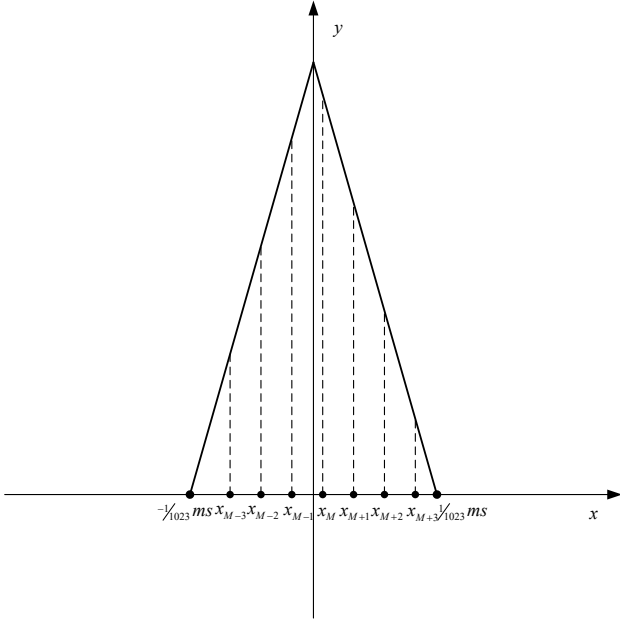


FIGURE 5: Autocorrelation peak values versus initial pseudorandom code phase shift.

Substitute the two sets of estimated amplitude  $A$  and noise variance  $\sigma'^2$ ,  $(\hat{A}_1, \hat{\sigma}'^2_1)$  and  $(\hat{A}_2, \hat{\sigma}'^2_2)$  into (52) and calculate  $\hat{x}(\hat{A}_1, \hat{\sigma}'^2_1)$  and  $\hat{x}(\hat{A}_2, \hat{\sigma}'^2_2)$ ; and the one, which approaches  $\bar{x}$  most, is taken as the estimated background power level.

## 5 Adaptive Change on Data Length of NCAC

From the former discussion, extension of NCAC length can elevate SNR effectively. However, the computation burden is heavy; hence, a tradeoff between increasing SNR and reducing

computation burden is necessary.

The ideal case the accumulation length can exactly guarantee the requirement on the SNR gain. To achieve this objective, two solutions are proposed and discussed of in this section: SNR gain formula-based method and probability formula inverse method.

In the SNR gain-based method, the SNR gain is depicted in

$$G(n) = 10 \log(n) - L(n) \quad (53)$$

where

$$L(n) = 10 \log \left[ \frac{1 + \sqrt{1 + 9.2n/D_c(1)}}{1 + \sqrt{1 + 9.2/D_c(1)}} \right] \quad (54)$$

$$D_c(1) = [\text{erfc}^{-1}(2P_{fa}) - \text{erfc}^{-1}(2P_d)]^2 \quad (55)$$

After coarse estimation of the amplitude,  $A$ , and noise variance,  $\sigma'^2$ , SNR can be calculated and the needed SNR gain can be determined which can furthermore fix the NCAC length.

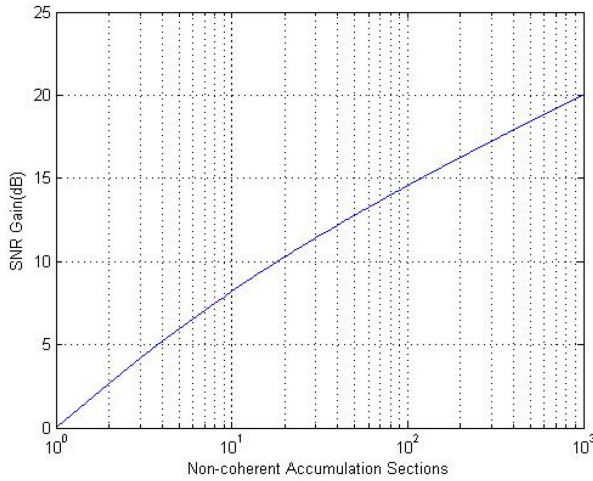
Figure 6 illustrates the SNR gain curve versus NCAC length, under the condition of  $P_{fa} = 10^{-7}$  and  $P_d = 0.9$ . SNR gain formula-based method is very easy. However, the estimated SNR is needed in this method. Indeed, before achieve successful GPS signal acquisition the estimated amplitude might be incorrect. Therefore, SNR gain-based method does not satisfy the requirement on accuracy of NCAC length. Hence, we introduce next the probability formula inverse method.

From (17) (see Appendix A), the probability of detection (PoD),  $P_d$ , and probability of PFA,  $P_{fa}$ , can be expressed as the function of accumulation length  $N$  and background noise power level,  $\sigma_x^2$ ,

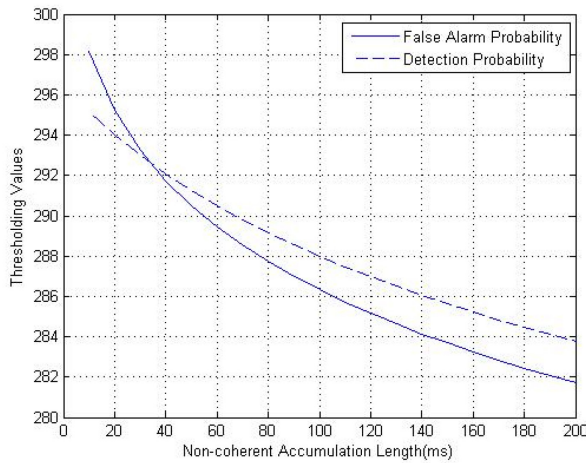
$$P_j = f_j(N, \sigma_x^2); \quad j = \{d, fa\} \quad (56)$$

PoD,  $P_d$ , and PFA,  $P_{fa}$ , are the expected GPS signal detection performance, which can be set as the expected values. Background noise power level,  $\sigma_x^2$ , can be estimated. Therefore, the NCAC length can be calculated by solving the combination of (56).

It is not straight forward to complex to solve the analytic solution of  $N$  from (56) by equating  $f_d(N, \sigma_x^2)$  with  $f_{fa}(N, \sigma_x^2)$ . Numeric method can also be employed to determine the NCAC length. Fortunately, the number  $N$  is not huge, which locates in the range from 200 to 400 in general cases. Therefore, to determine NCAC length  $N$  does not consume huge hardware and time resources.



(a) SNR gain caused by NCAC



(b) Principle of probability formula inverse method

FIGURE 6: (a) SNR gain caused by NCAC vs (b) principle of probability formula inverse method.

Figure 6 (b) illustrates the principle of probability formula inverse method, which is implemented by numeric method based on PoD of 0.9 ( $P_d = 0.9$ ) and PFA of  $10^{-6}$  ( $P_{fa} = 10^{-6}$ ).

The two curves correspond to the detection probability values and PFA values respectively, have a crossing point, whose x-coordinate equals to about 36 ms and the y-coordinate corresponds to threshold value.

## 6 Adaptive GPS Signal Detection Based On DDS

### 6.1 Principle of GPS Signal Detection Based on DDS

The study in the former sections focused on a single detection structure, which is employed in almost all traditional GPS receiver. When detecting GPS signal, FA and tracking loss will happen inevitably. As long as the degradation on GPS

signal occurs, the PFA and tracking loss increase.

Indeed, GPS receiver consumes a lot of time to re-arrive at successful acquisition state, after recovering from FA and tracking loss which leads to an increase of the average acquisition time (see Appendix B).

In this section, the application of DDS in GPS signal acquisition is discussed because of the obvious advantage of DDS lies in reducing the average time by lowering “penalty” time caused by FA and tracking loss.

DDS consists of a 2-tier GPS signal detection structure that works simultaneously because the probability of 2-tier detection structure encountering FA is much smaller than that of a single structure; i.e., one detection structure works effectively even if FA or tracking loss happens in the other structure. The 2-tier detection structures can be combined in parallel or cascaded form. Using the parallel detection structure, detection module *A* and detection module *B* work simultaneously to search for weak (or highly degraded) GPS signals. If one of the two detection modules (for example module *A*) claims that GPS signal has been detected, the other module (module *B*) exerts as the verification module to test whether FA happen in module *A*.

At the same time, module *A* still works to search for possible GPS signals in a GPS receiver. If module *B* finds that FA did not happen, GPS receiver turns to tracking state; if module *B* finds that FA happened in module *A*, GPS receiver can utilize the detection result of module *A*, while it searching.

Using the cascade detection structure, detection module *A* searches for weak (or degraded) GPS signals. Before module *A* claims that a weak GPS signal is detected, module *B* stays vacant. Only after module *A* has acquired a GPS signal, module *B* begins to verify.

By comparing and contrasting the two combination methods (or subsystems), the hardware resource in module *B* is not utilized. Therefore, parallel method (or subsystem) is preferred in this paper. Figure 7 illustrates the parallel DDS based GPS signal acquisition.

The traditional DDS is not a good solution to acquire GPS signal. The unfitness of conventional structure lies in two aspects: considering the discussions on GPS signal pseudorandom code carrier phase domain search in [40], the carrier frequency domain search was not been considered; the cascade form is studied in the existed double dwell acquisition structure.

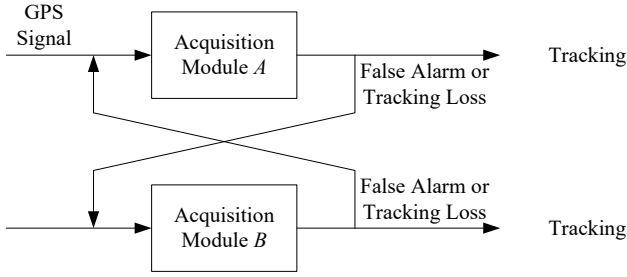


FIGURE 7: Block diagram of GPS signal acquisition based on double dwell structure.

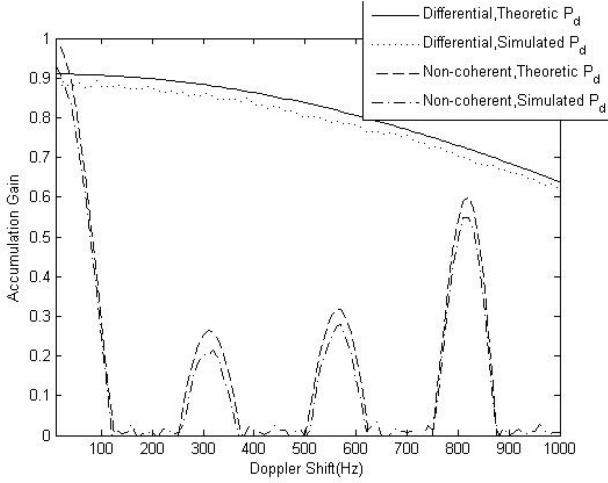


FIGURE 8: Accumulation gain versus Doppler shift.

CAC structure is taken as the first tier and NCAC as the second one. Such a structure can reduce average acquisition time effectively. In this section, the mixed parallel structure by DCAC and NCAC is proposed, which enables small probability of miss detection and reduce average acquisition time.

Since, NCAC and DCAC based GPS signal adaptive acquisition structures are previously discussed respectively; DCAC based GPS signal acquisition scheme possesses better Doppler shift tolerance than NCAC, which means detection statistics DCAC based acquisition scheme degrades less than NCAC based one at huge Doppler shift; on the contrary, when encountering small Doppler shift, NCAC based acquisition scheme can generate higher accumulation gain in SNR. The difference in detection performance is illustrated in Fig. 8. Also, DCAC based detection scheme is insensitive to data bit caused carrier phase transition.

Since of the advantages in different aspects of detection performance, DCAC and NCAC based GPS signal acquisition schemes are employed in the two modules respectively.

In the environment with huge Doppler shift, DCAC module can achieve coarse GPS signal acquisition; afterwards, NCAC

module performs accurate acquisition on GPS signal in the condition of small Doppler shift.

Figure 8 illustrates the different tolerance on Doppler shift between DCAC and NCAC.

From Fig. 8, the DCAC gain changes very slowly and smoothly while inducing less than 1.6 dB loss in the condition of 1-kHz Doppler shift. Since, NCAC gain fluctuates quickly, it is obvious that NCAC generates higher accumulation gain than DCAC in the condition of small Doppler shift. The simulation result verified the advantages in different aspects of detection performance.

Equations derived in the DDS Markov chain analysis (see Appendix B) are used to produce the numerical results given in the following section.

## 7 Numerical, Theoretical Results

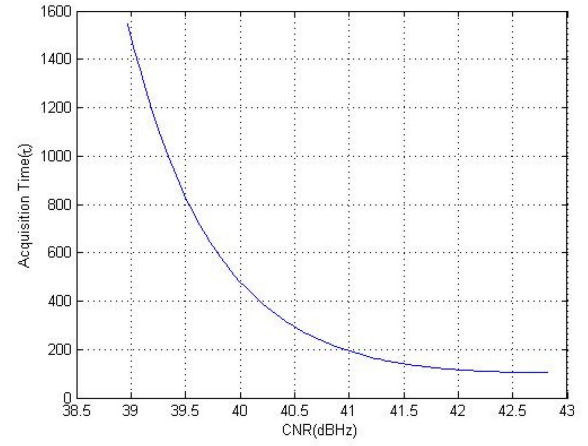
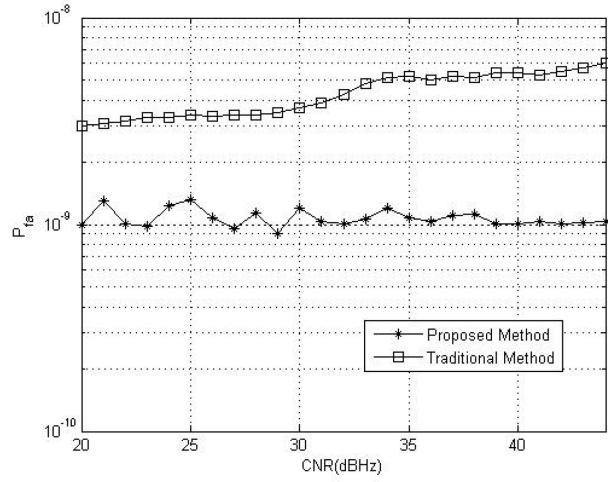
To test the effectiveness of the methods proposed in this paper, experiments and simulations are performed in this section. Since background noise power level estimation is fundamental in adaptive GPS signal processing, the improved noise level estimation in GPS signal is initially performed.

Place a GPS antenna in an open space, where GPS signal can directly arrive at receiver. Personal computer is connected to the front end of GPS receiver to record signals from the expected GPS satellite ( $SVN = 3$ ). In contrast, traditional background noise power level estimation method is employed.

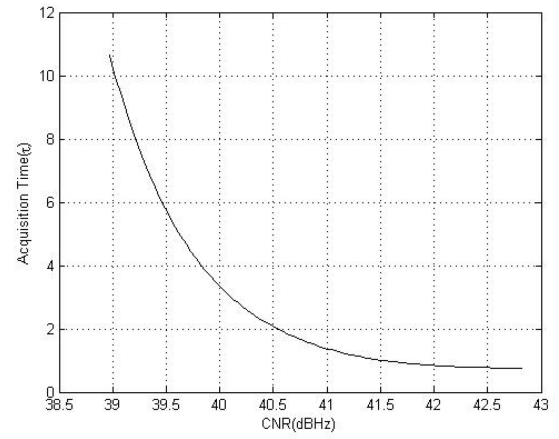
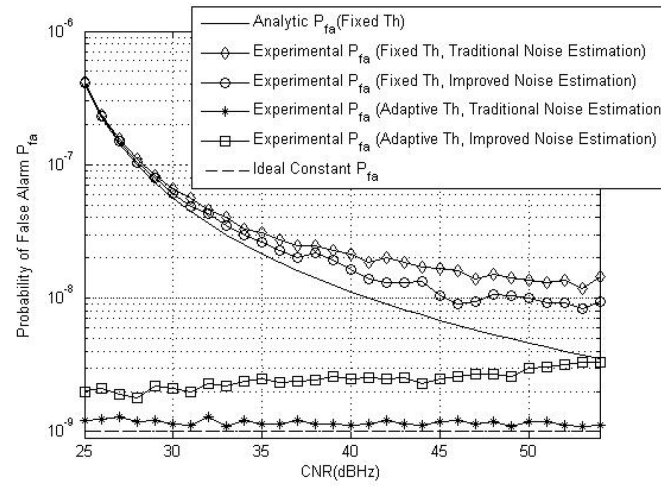
Using the improved and traditional estimation method, two background noise levels are calculated respectively; afterwards, two threshold values calculated by numeric method in the condition of  $10^{-9}$ ; in the time duration when  $SVN = 3$  GPS satellite is invisible, gather data with GPS antenna and store the data in computer. To test detection performance at different CNR, white Gaussian noise is posed on the received GPS signal to calculate PFA continuously. Figure 9 demonstrates the PFA curve. From Fig. 9 (a), using traditional background noise level estimation method, the actual rate of FA is higher than the rate for the improved method.

The difference in detection performance is caused by the non-zero cross-correlation result, which will be taken as noise in traditional method.

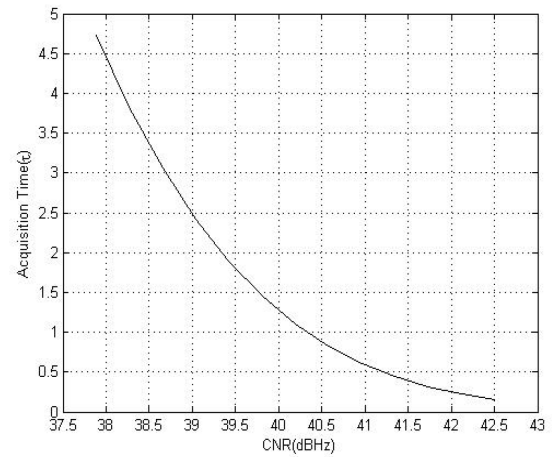
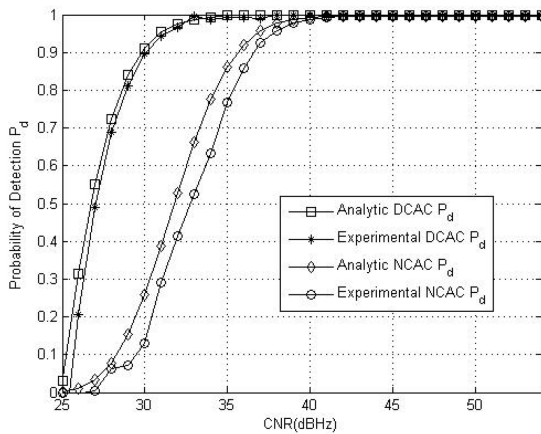
To test GPS signal detection performance based on DCAC, simulations are performed in this section. To demonstrate the advantage of improved noise estimation method, traditional noise estimation method is also used to determine threshold.



(a) Pseudorandom code phase at one frequency bin



(b) Doppler shift

FIGURE 9: PFA curve versus CNR.<sup>3</sup>

(c) Double dwell structure

FIGURE 10: Detection probability curves versus CNR.

FIGURE 11: Average acquisition time for NCAC based GPS signal acquisition scheme for searching by step: (a) pseudorandom code phase at one frequency bin; (b) Doppler shift; (c) double dwell structure.

<sup>3</sup> For this plot  $T \equiv \varrho$  or  $T \equiv \rho$ .

To test the advantage of adaptive threshold setting, GPS detection scheme based on fixed threshold is performed in contrast. To reflect detection performance at different CNR continuously, simulated GPS data is used which is generated in the simulation computer.

From the Fig. 9 (b), the CFAR GPS detection scheme based on DCAC brings in smaller PFA than detection scheme with fixed threshold; furthermore, the detection performance with improved noise estimation method outperforms the traditional noise estimation method. From the Fig. 9 (b), using traditional noise estimation method, the PFA increases when CNR for unexpected GPS signal becomes larger. Since of the non-zero cross-correlation result, unexpected GPS signal will generate larger interference to expected GPS signal when CNR increase.

Test is also performed to make a contrast on the detection performance between DCAC and NCAC based scheme. To illustrate the advantage of DCAC scheme on Doppler tolerance, GPS signal at large Doppler shift (500 Hz) is generated in the test. Figure 10 illustrates the detection performance by detection probability.

From Fig. 10, DCAC based scheme approach 0.9 PoD at about 30 dB-Hz, while NCAC based scheme need the CNR at about 36 dB-Hz.

To illustrate the advantage of DDS based detection scheme, the average time of acquiring GPS signal is calculated. As contrast, the average acquisition time curve of DCAC based detection scheme is also presented.

Figure 11 (a) illustrates the average acquisition time for searching in the domain of pseudorandom code phase, using NCAC based GPS signal acquisition scheme, which is very time consuming. Indeed, the acquisition time curve denotes the result at one Doppler frequency bin. To cover all Doppler shift bins, ten times of the average acquisition time is needed. Fortunately, since circular correlation implemented by FFT is widely used in GPS signal acquisition, only searching in Doppler shift domain is needed. Figure 11 (b) demonstrates the average acquisition time curve for GPS signal searching in Doppler shift domain. Compared with the average acquisition time consumed in searching in pseudorandom code domain, time for searching in Doppler shift domain is much shorter.

As mentioned in previous section, the traditional single dwell-based GPS acquisition scheme might encounter penalty in processing time because of FA. To reduce processing time caused by FA, DDS employed to acquire GPS signal. The average acquisition time curve for DDS based GPS acquisition

scheme is plotted in Fig. 11 (c).

From Fig. 11 (c), time consumed in DDS based GPS signal acquisition scheme decreases to about one half of the average acquisition time in Fig. 11 (b). The DDS based GPS signal acquisition scheme offers to main advantage. The 2-tier GPS signal detection module reduces the probability of FA and tracking loss. Consequently, the time penalty from FA and tracking loss decreases significantly; since the larger the Doppler shift tolerance for DCAC scheme, the wider the DDS based GPS acquisition range that includes the entire possible Doppler shift by frequency bins, compared with NCAC based single acquisition structure.

## 8 Conclusions

To improve GPS signal acquisition performance, adaptive schemes are employed in GPS receiver signal processing acquisition design. Gaussian, CAC, NCAC, and DCAC GPS signal detection structures based on CFAR are presented in this paper. For this adaptive structure to work, the analysis suggests that the estimation of the background noise power level is very important. Therefore, improved noise background noise estimation method is proposed, which considers the non-zero cross-correlation result of GPS signal.

As for the detection performance on average time, two aspects of work have been studied. First, the adaptive determination on accumulation length is proposed; second, DDS based GPS signal detection scheme is studied. The DDS utilizes the high gain of NCAC at small Doppler shift and high Doppler shift tolerance for DCAC. In the final part, simulations have been performed to test the effectiveness of improvement on average acquisition time.

The adaptive structure possesses the advantage on stable detection performance; furthermore, average acquisition time is reduced since of the adaptive accumulation length and less PFA. The improvement is very important for GPS signal detection in challenging environments.

## 9 Acknowledgement

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## 11 Appendix A: Probability of Misdetec-tion

Under the assumption that,  $p(x''|H_1)$  follows a Gaussian distribution the existence of GPS signal is determined by the probability of miss detection as

$$P_{md} = \int_{-\infty}^q p(x''|H_1) dx'' = F(q) \quad (57)$$

which can be written as

$$P_{md} = \int_0^{q-A} \frac{1}{\sqrt{2\pi}} e^{-0.5(x'-A)^2} dx' = \begin{cases} \text{erf}(q-A) & q \geq A \\ 0 & q < A \end{cases} \quad (58)$$

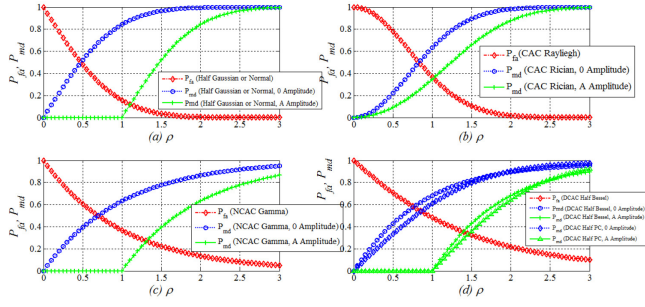


FIGURE 12: Numerical calculations of  $P_{fa}$  (red),  $P_{md}$  (blue  $A = 0$ , green  $A = 1$ ): (a) Gaussian or Normal; (b) CAC or Rician; (c) NCAC or Gamma (d) half Bessel and parabolic cylinder for  $A = \{0,1\}$ .

Under the assumption that,  $p(x'_a|H_1)$  follows a Rice distribution, using the substitutions,  $x''_a \equiv x'_a$ ,  $\mu''_{x|H_1} \equiv \frac{\mu'_{x|H_1}}{\sigma'_x}$ , the following is obtained

$$p(x''_a|H_1) = \frac{x''_a}{\sigma'_x} e^{-\frac{x''_a{}^2 + (A^2 \equiv \mu''_{x|H_1}{}^2)}{2}} I_0(x''_a \mu''_{x|H_1}) \quad (59)$$

Hence,  $P_{md}$  can be written as

$$P_{md} = \int_0^Q p(x''_a|H_1) dx''_a = F_{Rice}(Q, 1, A) + F_{Rice}(0, 1, A) \quad (60)$$

Since the Rice cdf is given by

$$F_{Rice}(Q, 1, A) = 1 - Q_1(A, x''_a) \quad (61)$$

where  $Q_1(a, b)$  is the Marcum Q-function [1].

Under the assumption that  $p(z'|H_1)$  follows a Gamma distribution then we have

$$p(z'|H_1) = \frac{(z'-A)^{M-1} e^{-0.5(z'-A)}}{2^M \Gamma(M)} \quad (62)$$

Hence, the probability of misdetection is

$$P_{md} = \int_0^{Q-A} \frac{(z'-A)^{M-1} e^{-0.5(z'-A)} dz'}{2^M \Gamma(M)} = \begin{cases} Q(Q-A) & Q \geq A \\ 0 & Q < A \end{cases} \quad (63)$$

Under the assumption that  $p(z'|H_1)$  follows a generalized Bessel distribution of the second kind then we have

$$p(z'|H_1) = \frac{\pi^{-\frac{1}{2}}(d+1)^{2p+1} \Gamma(p+\frac{3}{2}) |z'-A|^p e^{-\frac{d}{2}|z'-A|} K_p\left(\frac{|z'-A|}{a}\right)}{2^{(p+1)} \Gamma(2p+1) a^{p+1} F\left(2p+1, p+\frac{1}{2}, p+\frac{3}{2}, \frac{d-1}{2d+1}\right)} \quad (64)$$

Hence, the probability of misdetection is

$$P_{md} = \int_0^{Q-A} \frac{\pi^{-\frac{1}{2}}(d+1)^{2p+1} \Gamma(p+\frac{3}{2}) |z'-A|^p e^{-\frac{d}{2}|z'-A|} K_p\left(\frac{|z'-A|}{a}\right)}{2^{(p+1)} \Gamma(2p+1) a^{p+1} F\left(2p+1, p+\frac{1}{2}, p+\frac{3}{2}, \frac{d-1}{2d+1}\right)} dz' \quad (65)$$

Finally, under the assumption that  $p(z'|H_1)$  follows a parabolic cylinder distribution then we have

$$p(z'|H_1) = \frac{\sqrt{2} d_0 \sum_{k=0}^{N-1} H_k(N)(N-k-1)! [b_0 D_{k-N}(c_0) + b_1 D_{k-N}(c_1)]}{a^{k-N}} \quad (66)$$

$$4\sqrt{\pi} e^{3c^4} \left(\frac{2a}{\sqrt{3}}\right)^N a \sum_{k=0}^{N-1} H_k(N) n! \left(\frac{\sqrt{3}}{2a}\right)^k D_{k-N}\left(\frac{2a}{\sqrt{3}c^2}\right)$$

Hence, the probability of misdetection is

$$P_{md} = \int_0^{Q'} \frac{\sqrt{2} d_0 \sum_{k=0}^{N-1} H_k(N)(N-k-1)! [b_0 D_{k-N}(c_0) + b_1 D_{k-N}(c_1)]}{a^{k-N}} dz' \quad (67)$$

$$4\sqrt{\pi} e^{3c^4} \left(\frac{2a}{\sqrt{3}}\right)^N a \sum_{k=0}^{N-1} H_k(N) n! \left(\frac{\sqrt{3}}{2a}\right)^k D_{k-N}\left(\frac{2a}{\sqrt{3}c^2}\right)$$

where  $Q' = Q - A$

Equations (57) through (67) have been employed to produce the results of Fig. 15.

Figure 12 shows various calculations of  $P_{fa}$  (red),  $P_{md}$  (blue  $A = 0$ , green  $A = 1$ ) for (a) Gaussian or Normal; (b) CAC or Rician; (c) NCAC or Gamma (d) half Bessel and parabolic cylinder for  $A = \{0,1\}$ . As shown in Fig. 12 (a) through (b) in terms of  $P_{fa}$  and  $P_{md}$  Gaussian is the best, then NCAC or Gamma, then Bessel or Parabolic cylinder, then CAC or Rician.

Nevertheless, we have to keep in mind that Bessel or parabolic cylinder model DCAC under severe indoor multipath channel modeling. SO, even under these adverse conditions Bessel or Parabolic cylinder model DCAC have a respectable performance in terms of  $P_{fa}$  and  $P_{md}$ .

It is also important to show that the generalized Bessel of the second kind and or parabolic cylinder have comparable performance. As a summary, this is probably the only paper that offers these kinds of analyses and these kinds of details not found in any other paper or publication in part or as a whole.

## 12 Appendix B: DDS Markov Chain Analyses

DDS Markov Chain analyses consist of: (1) state transition diagram of DDS based on GPS signal acquisition and (2) average acquisition time.

### 12.1 State Transition Diagram of DDS Based on GPS Signal Acquisition

Using DDS, GPS receiver can be classified into three distinct states:

1. Neither DCAC based detection scheme has acquired a GPS signal, nor NCAC based detection scheme does confirm the existence of GPS signal;
2. DCAC based detection scheme claims that a GPS signal is acquired, but NCAC based detection scheme does not confirm the existence of a GPS signal;

3. DCAC based detection scheme claims that a GPS signal is acquired, and NCAC based detection scheme confirms the existence (or presence) of a GPS signal.

To illustrate the stochastic model describing the state transition relationship among these three states, a Markov chain<sup>v</sup> is given in Fig. 13.

Where  $P_{d_D}$  and  $P_{d_N}$  denote PoD of DCAC and NCAC scheme respectively;  $P_{fa_D}$  and  $P_{fa_N}$  denote PFA scheme respectively.

Suppose  $z$  denotes the regular time duration for one time of correct judgment on GPS signal processing. And one time of FA incur  $k$  times the regular time duration.

Using Mason chain formula [41], the transition function can be derived from Markov chain, which is presented in

$$H(z) = H_1(z) \cdot H_2(z) \cdot H_3(z) \quad (68)$$

$$H_1(z) = \frac{1}{1 - (1 - P_{fa_D})z - (1 - P_{d_D})z} \quad (69)$$

$$H_2(z) = \frac{1}{(1 - P_{d_N})z} \quad (70)$$

$$H_3(z) = \frac{\text{Num}[H_3(z)]}{\text{Den}[H_3(z)]} \quad (70)$$

Where

$$\text{Num}[H_3(z)] = (P_{d_D}z + P_{fa_D}z^k)(P_{d_N}z + P_{fa_N}z^k) \quad (71)$$

$$\text{Den}[H_3(z)] = 1 - a(z)z - b(z)z \quad (72)$$

Where

$$a(z) = (P_{d_D}z + P_{fa_D}z^k)(1 - P_{d_N}) \quad (73)$$

$$b(z) = (P_{d_D}z + P_{fa_D}z^k)(P_{d_N}z + P_{fa_N}z^k)(1 - P_{fa_N}) \quad (74)$$

This concludes the discussion of the state transition diagram of DDS based on GPS signal acquisition.

## 12.2 Average Acquisition Time

To calculate the average acquisition time of DDS based GPS signal acquisition structure, calculate the partial derivative of the natural logarithm for  $H(z)$ , calculate partial derivative of  $\log(H(z))$  versus  $z$ ,

$$\frac{\partial \log(H(z))}{\partial z} = A(z) + B(z) + C(z) + D(z) + E(z) + F(z) \quad (75)$$

where

$$A(z) = \frac{P_{d_D} + k \cdot P_{fa_D} z^{k-1}}{P_{d_D}z + P_{fa_D}z^k} \quad (76)$$

$$B(z) = \frac{P_{d_N} + k \cdot P_{fa_N} z^{k-1}}{P_{d_N}z + P_{fa_N}z^k} \quad (77)$$

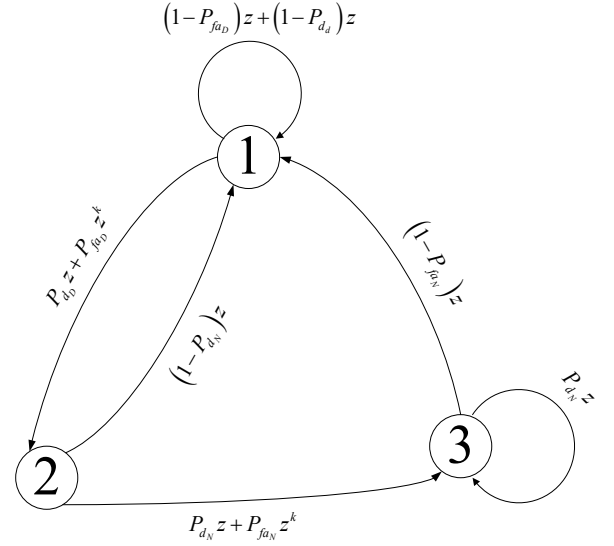


FIGURE 13: DDS Markov chain of GPS receiver.

$$C(z) = \frac{P_{d_N}}{1 - P_{d_N}z} \quad (78)$$

$$D(z) = \frac{2 - P_{fa_D} - P_{d_D}}{1 - (1 - P_{fa_D})z - (1 - P_{d_D})z} \quad (79)$$

$$E(z) = \frac{\text{Num}[E(z)]}{\text{Den}[E(z)]} \quad (80)$$

$$\text{Num}[E(z)] = \text{Num}_A[E(z)] + \text{Num}_B[E(z)] \quad (81)$$

$$\text{Den}[E(z)] = 1 - \text{Den}_A[E(z)] - \text{Den}_B[E(z)] \quad (82)$$

$$F(z) = \frac{\text{Num}[F(z)]}{\text{Den}[F(z)]} \quad (83)$$

$$\text{Num}[F(z)] = \text{Num}_A[F(z)] + \text{Num}_B[F(z)] \quad (84)$$

$$\text{Den}[F(z)] = 1 - \text{Den}_A[F(z)] - \text{Den}_B[F(z)] \quad (85)$$

where

$$\text{Num}_A[E(z)] = (P_{d_D}z + P_{fa_D}z^k)(1 - P_{d_N}) \quad (86)$$

$$\text{Num}_B[E(z)] = (1 - P_{d_N})(P_{d_D} + kP_{fa_D}z^{k-1})z \quad (87)$$

Next, let us determine  $\text{Den}_A[E(z)]$  and  $\text{Den}_B[E(z)]$

$$\text{Den}_A[E(z)] = (P_{d_D}z + P_{fa_D}z^k)(1 - P_{d_N})z \quad (88)$$

$$\text{Den}_B[E(z)] = \text{Den}_{B1}[E(z)]\text{Den}_{B2}[E(z)] \quad (89)$$

where

$$\text{Den}_{B1}[E(z)] = (P_{d_D}z + P_{fa_D}z^k)(P_{d_N}z + P_{fa_N}z^k) \quad (90)$$

$$\text{Den}_{B2}[E(z)] = (1 - P_{fa_N})z \quad (91)$$

Next, let us compute  $\text{Num}_A[F(z)]$

$$\text{Num}_A[F(z)] = \text{Num}_{A1}[F(z)]\text{Num}_{A2}[F(z)] \quad (92)$$

Where

$$\text{Num\_A1}[F(z)] = (P_{d_D}z + P_{fa_D}z^k)(P_{d_N}z + P_{fa_N}z^k) \quad (93)$$

$$\text{Num\_A2}[F(z)] = 1 - P_{fa_N} \quad (94)$$

Next, let us compute,  $\text{Num\_B}[F(z)]$

$$\text{Num\_B}[F(z)] = \text{Num\_B1}[F(z)]\text{Num\_B2}[F(z)] \quad (95)$$

Where

$$\text{Num\_B1}[F(z)] = (P_{d_N}z + P_{fa_N}z^k)(1 - P_{fa_N}) \quad (96)$$

$$\text{Num\_B2}[F(z)] = (P_{d_D} + P_{fa_D}z^{k-1})z \quad (96)$$

Next, let us calculate  $\text{Den\_A}[F(z)]$

$$\text{Den\_A}[F(z)] = (P_{d_D}z + P_{fa_D}z^k)(1 - P_{d_N})z \quad (97)$$

Next, let us determine  $\text{Den\_B}[F(z)]$

$$\text{Den\_B}[F(z)] = \text{Den\_B1}[F(z)]\text{Den\_B2}[F(z)] \quad (98)$$

Where

$$\text{Den\_B1}[F(z)] = (P_{d_D}z + P_{fa_D}z^k)(P_{d_N}z + P_{fa_N}z^k) \quad (99)$$

$$\text{Den\_B2}[F(z)] = (1 - P_{fa_N})z \quad (100)$$

Therefore, the average acquisition time can be calculated as,

$$\bar{\tau} = A(1) + B(1) + C(1) + D(1) + E(1) + F(1) \quad (101)$$

$$\bar{\tau} = A(1) + B(1) + C(1) + D(1) + E(1) + F(1) \quad (102)$$

Where

$$A(1) = \frac{P_{d_D} + kP_{fa_D}}{P_{d_D} + P_{fa_D}} \quad (103)$$

$$B(1) = \frac{P_{d_N} + kP_{fa_N}}{P_{d_N} + P_{fa_N}} \quad (104)$$

$$C(1) = \frac{P_{d_N}}{1 - P_{d_N}} \quad (105)$$

$$D(1) = \frac{2 - P_{fa_D} - P_{d_D}}{P_{fa_D} + P_{d_D} - 1} \quad (106)$$

Next, let us compute  $\text{Num\_A}[E(1)]$  and  $\text{Num\_B}[E(1)]$

$$\text{Num\_A}[E(1)] = (1 - P_{d_N})(P_{d_D} + P_{fa_D}) \quad (107)$$

$$\text{Num\_B}[E(1)] = (1 - P_{d_N})(P_{d_D} + kP_{fa_D}) \quad (108)$$

Hence,

$$\text{Num}[E(1)] = (1 - P_{d_N})[P_{d_D} + (1 + k)P_{fa_D}] \quad (109)$$

Next,

$$\text{Den\_A}[E(1)] = (P_{d_D} + P_{fa_D})(1 - P_{d_N}) \quad (110)$$

Next, let us calculate  $\text{Den\_B1}[E(1)]$  and  $\text{Den\_B2}[E(1)]$

$$\text{Den\_B1}[E(1)] = (P_{d_D} + P_{fa_D})(P_{d_N} + P_{fa_N}) \quad (111)$$

$$\text{Den\_B2}[E(1)] = 1 - P_{fa_N} \quad (112)$$

Next, let us determine  $\text{Num\_A1}[F(1)]$  and  $\text{Num\_A2}[F(1)]$

$$\text{Num\_A1}[F(1)] = (P_{d_D} + P_{fa_D})(P_{d_N} + P_{fa_N}) \quad (113)$$

$$\text{Num\_A2}[F(1)] = 1 - P_{fa_N} \quad (114)$$

Hence,

$$\text{Num\_A}[F(1)] = \text{Num\_A1}[F(1)]\text{Num\_A2}[F(1)] \quad (115)$$

Next, let us compute  $\text{Num\_B1}[F(1)]$  and  $\text{Num\_B2}[F(1)]$

$$\text{Num\_B1}[F(1)] = (P_{d_N} + P_{fa_N})(1 - P_{fa_N}) \quad (116)$$

$$\text{Num\_B2}[F(1)] = (P_{d_D} + P_{fa_D}) \quad (117)$$

Hence,

$$\text{Num\_B}[F(1)] = \text{Num\_B1}[F(1)]\text{Num\_B2}[F(1)] \quad (118)$$

Next,

$$\text{Den\_A}[F(1)] = (P_{d_D} + P_{fa_D})(1 - P_{d_N}) \quad (119)$$

Next, let us calculate  $\text{Den\_B1}[F(1)]$  and  $\text{Den\_B2}[F(1)]$

$$\text{Den\_B1}[F(1)] = (P_{d_D} + P_{fa_D})(P_{d_N} + P_{fa_N}) \quad (119)$$

$$\text{Den\_B2}[F(1)] = (1 - P_{fa_N}) \quad (120)$$

Hence,

$$\text{Den\_B}[F(1)] = \text{Den\_B1}[F(1)]\text{Den\_B2}[F(1)] \quad (121)$$

This concludes the discussion on average acquisition time and Appendix B DDS Markov chain analyses.

<sup>i</sup> Further discussion on the computation of the probability of detection or miss detection is given in Appendix A.

<sup>ii</sup> Ditto

<sup>iii</sup> Ditto

<sup>iv</sup> Ditto

<sup>v</sup> A Markov chain is a stochastic model describing a sequence of possible events in which the probability of each event depends only on the state attained in the previous event.